Instructions

• Use black ink or ball-point pen.
• Answer all questions.
• Answer the questions in the spaces provided – there may be more space than you need.
• Diagrams are NOT accurately drawn, unless otherwise indicated.
• You must show all your working out.

Information

• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end
1. The equation $x^3 + 7x - 2 = 55$ has a solution between 3 and 4.

   Use trial and improvement to find this solution.
   Give your answer to 1 decimal place.
2. Use trial and improvement to solve \( x^3 - x^2 = 85 \)

Give your answer to 1 decimal place.
3. Use trial and improvement to solve $x^3 + 5x = 70$

Give your answer to 1 decimal place.
4. An approximate solution to an equation is found using this iterative process:

\[ x_{n+1} = \sqrt{x_n} + 10 \quad \text{and} \quad x_1 = 3 \]

a) Work out the values of \( x_2 \) and \( x_3 \)

\[ \text{..................} \quad \text{...............} \quad (2) \]

b) Work out the solution to 3 decimal places

\[ \text{..................} \quad (1) \]
5. An approximate solution to an equation is found using this iterative process:

\[ x_{n+1} = \frac{(x_n)^3 - 3}{8} \quad \text{and} \quad x_1 = -1 \]

a) Work out the values of \( x_2 \) and \( x_3 \)

\[ \ldots \quad \ldots \quad (2) \]

b) Work out the solution to 6 decimal places

\[ \ldots \quad (1) \]
6. A sequence is defined by the term-to-term rule:

\[ U_{n+1} = U_n^2 - 8U_n + 17 \]

\[ \text{a) Given that } U_1 = 4, \text{ find } U_2 \text{ and } U_3 \]

\[ \text{.................. .................. (2)} \]

\[ \text{b) Given instead that } U_1 = 2, \text{ find } U_2, U_3 \text{ and } U_{100} \]

\[ \text{.................. .................. .................. (3)} \]
7. (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x=0$ and $x=1$.

(b) Show that the equation $x^3 + 4x = 1$ can be rearranged to give $x = \frac{1}{4} - \frac{x^3}{4}$.

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice, to find an estimate to the solution of $x^3 + 4x = 1$. 

............ (2)

............ (1)

............ (3)