GCSE (1 – 9)
Compound and Inverse Functions

Instructions

• Use black ink or ball-point pen.
• Answer all questions.
• Answer the questions in the spaces provided
  – there may be more space than you need.
• Diagrams are NOT accurately drawn, unless otherwise indicated.
• You must show all your working out.

Information

• The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end.
1. Given that $f(x) = x - 4$ find:
   a) $f(5)
   \begin{align*}
f(5) &= 5 - 4 \\
     &= 1 \quad \cdots (1)
   \end{align*}
   
   b) $f(3)$
   \begin{align*}
f(3) &= 3 - 4 \\
     &= -1 \quad \cdots (1)
   \end{align*}

2. Given that $g(x) = 2x^2 - 10$ find:
   a) $g(2)$
   \begin{align*}
g(2) &= 2(2)^2 - 10 \\
     &= 8 - 10 \\
     &= -2 \quad \cdots (1)
   \end{align*}
   
   b) $g(-2)$
   \begin{align*}
g(-2) &= 2(-2)^2 - 10 \\
     &= 8 - 10 \\
     &= -2 \quad \cdots (1)
   \end{align*}
   
   c) Solve: $g(x) = 8$
   \begin{align*}
2x^2 - 10 &= 8 \\
2x^2 &= 18 \\
x^2 &= 9
   \end{align*}
   \begin{align*}
x &= \pm 3 \quad (3)
   \end{align*}
3. Given that \( f(x) = 3x - 5 \) find:

a) \[ f(3) = 3(3) - 5 = 9 - 5 = 4 \]  \hspace{1cm} \text{(1)}

b) \[ f(-2) = 3(-2) - 5 = -6 - 5 = -11 \]  \hspace{1cm} \text{(1)}

c) \text{Solve: } f(x) = 1

\[ 3x - 5 = 1 \]

\[ 3x = 6 \]

\[ x = 2 \]  \hspace{1cm} \text{(2)}

4. Given that \( f(x) = x^2 - 3 \) find:

a) \[ f(10) = (10)^2 - 3 = 100 - 3 = 97 \]  \hspace{1cm} \text{(1)}

b) \[ f(-1) = (-1)^2 - 3 = 1 - 3 = -2 \]  \hspace{1cm} \text{(1)}

c) \text{Find: } f^{-1}(x)

\[ y = x^2 - 3 \]

\[ y + 3 = x^2 \]

\[ \sqrt{y + 3} = x \]

\[ f^{-1}(x) = \sqrt{x + 3} \]  \hspace{1cm} \text{(2)}
5. Given that \( f(x) = 2x - 4 \) and \( g(x) = 3x + 5 \)

   a) Find: \( gf(3) \)

   \[
   f(3) = 2(3) - 4 \\
   = 6 - 4 \\
   = 2
   \]

   \[
   g(2) = 3(2) + 5 \\
   = 6 + 5
   \]

   \[
   \dot{\ldots\ldots} \quad (2)
   \]

   b) Work out an expression for: \( f^{-1}(x) \)

   \[
   y = 2x - 4
   \]

   \[
   y + 4 = 2x
   \]

   \[
   \frac{1}{2}(y+4) = x
   \]

   \[
   f^{-1}(x) = \frac{1}{2}(x+4)
   \]

   \[
   f^{-1}(x) = \frac{1}{2}(x+4) \quad (2)
   \]

   c) Solve: \( f(x) = g(x) \)

   \[
   2x - 4 = 3x + 5
   \]

   \[
   -4 = x + 5
   \]

   \[
   x = -9
   \]

   \[
   \dot{\ldots\ldots} \quad (2)
   \]
6. Given that \( f(x) = 3x + 1 \) and \( g(x) = x^2 \)

   a) Write down an expression for: \( fg(x) \)

   \[
   3x^2 + 1 \quad (2)
   \]

   b) Work out an expression for: \( gf(x) \)

   \[
   (3x+1)^2 \quad (2)
   \]

   c) Solve: \( fg(x) = gf(x) \)

   \[
   3x^2 + 1 = (3x + 1)^2
   
   3x^2 + 1 = 9x^2 + 6x + 1
   
   0 = 6x^2 + 6x
   
   0 = x^2 + x
   
   0 = x(x + 1)
   
   x = 0, x = -1 \quad (3)
   \]
7. Given that \( f(x) = x^2 - 17 \) and \( g(x) = x + 3 \)

a) Work out an expression for: \( g^{-1}(x) \)

\[
y = x + 3
\]

\[
y - 3 = x
\]

\[
g^{-1}(x) = x - 3 \quad (2)
\]

b) Work out an expression for: \( f^{-1}(x) \)

\[
y = x^2 - 17
\]

\[
y + 17 = x^2
\]

\[
\sqrt{y + 17} = x
\]

\[
f^{-1}(x) = \sqrt{x + 17} \quad (2)
\]

c) Solve: \( f^{-1}(x) = g^{-1}(x) \)

\[
\sqrt{x + 17} = x - 3
\]

\[
x + 17 = (x - 3)^2
\]

\[
x + 17 = x^2 - 6x + 9
\]

\[
0 = x^2 - 7x - 8
\]

\[
0 = (x - 8)(x + 1)
\]

\[
x = 8 \quad x = -1
\]

\[
\text{...............} \quad (4)
\]
8. A function $f$ is defined such that

$$f(x) = x^2 - 1$$

a) Find an expression for $f(x-2)$

$$f(x-2) = (x-2)^2 - 1$$

$$= x^2 - 2x - 2x + 4 - 1$$

$$= x^2 - 4x + 3$$

$$\therefore x^2 - 4x + 3 \quad (2)$$

b) Hence solve $f(x-2) = 0$

$$x^2 - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$

$x = 3$  \hspace{0.5cm}  $x = 1$

$$x = 3, \quad x = 1 \quad (2)$$
9. A function \( f \) is defined such that
\[
f(x) = 4x - 1
\]
a) Find: \( f^{-1}(x) \)
\[
y = 4x - 1
\]
\[
y + 1 = 4x
\]
\[
\frac{y + 1}{4} = x
\]
\[
f^{-1}(x) = \frac{x + 1}{4}
\]

\[\text{(2)}\]

The function \( g \) is such that
\[
g(x) = kx^2 \quad \text{where} \quad k \text{ is a constant}
\]
Given that \( fg(2) = 12 \)

b) Work out the value of \( k \)
\[
g(2) = k(2)^2
\]
\[
= 4k
\]
\[
f(4k) = 4(4k) - 1
\]
\[
= 16k - 1
\]
\[
16k - 1 = 12
\]
\[
16k = 13
\]
\[
k = \frac{13}{16}
\]

\[\text{(2)}\]