Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Factor Theorem and

Remainder Theorem

Materials required for examination
Mathematical Formulae (Pink or Green)

Items included with question papers
Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. $f(x) = 2x^3 + x^2 - 5x + c$, where c is a constant.

Given that $f(1) = 0$,

(*a*) find the value of c,

(2)

(*b*) factorise $f(x)$ completely,

(4)

(*c*) find the remainder when $f(x)$ is divided by $(2x - 3)$.

(2)

1a) $f(1) = 0$

$2(1)^3 + (1)^2 - 5(1) + c = 0$

$2 + 1 - 5 + c = 0$

$-2 + c = 0$

$c = 2$

b) $2x^3 + x^2 - 5x + 2$ $\quad$ [$f(1) = 0 \therefore (x-1)$ is a factor]

$$
\begin{array}{r}
2x^2 + 3x - 2 \\
x - 1 \,\big)\, 2x^3 + x^2 - 5x + 2 \\
2x^3 - 2x^2 \\
3x^2 - 5x \\
3x^2 - 3x \\
-2x + 2 \\
-2x + 2 \\
\hline
0
\end{array}
$$

$(x - 1)(2x^2 + 3x - 2)$

$(x - 1)(2x - 1)(x + 2)$

c) $f(1.5) = 2(1.5)^3 + (1.5)^2 - 5(1.5) + 2$

$= 7/2$

2. $$f(x) = 3x^3 - 5x^2 - 16x + 12.$$

(*a*) Find the remainder when f(*x*) is divided by $(x - 2)$. **(2)**

Given that $(x + 2)$ is a factor of f(*x*),

(*b*) factorise f(*x*) completely. **(4)**

a/ $f(2) = 3(2)^3 - 5(2)^2 - 16(2) + 12$

$= \underline{\underline{-16}}$

b/

$$\begin{array}{r|l} & 3x^2 - 11x + 6 \\ \hline x+2 & 3x^3 - 5x^2 - 16x + 12 \\ & 3x^3 + 6x^2 \\ & \quad -11x^2 - 16x \\ & \quad -11x^2 - 22x \\ & \qquad 6x + 12 \\ & \qquad 6x + 12 \\ \hline & \qquad\quad 0 \end{array}$$

$(x+2)(3x^2 - 11x + 6)$

$(x+2)(3x-2)(x-3)$

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3. (*a*) Use the factor theorem to show that $(x + 4)$ is a factor of $2x^3 + x^2 - 25x + 12$. **(2)**

(*b*) Factorise $2x^3 + x^2 - 25x + 12$ completely. **(4)**

a/ $f(-4) = 2(-4)^3 + (-4)^2 - 25(-4) + 12$

$= 0$

b/

$$\begin{array}{r|l} & 2x^2 - 7x + 3 \\ \hline x+4 & 2x^3 + x^2 - 25x + 12 \\ & 2x^3 + 8x^2 \\ & \quad -7x^2 - 25x \\ & \quad -7x^2 - 28x \\ & \qquad 3x + 12 \\ & \qquad 3x + 12 \\ \hline & \qquad\quad 0 \end{array}$$

$(x+4)(2x^2 - 7x + 3)$

$\underline{(x+4)(2x-1)(x-3)}$

4. $$f(x) = 2x^3 + 3x^2 - 29x - 60.$$

(*a*) Find the remainder when f(x) is divided by (x + 2). **(2)**

(*b*) Use the factor theorem to show that (x + 3) is a factor of f(x). **(2)**

(*c*) Factorise f(x) completely. **(4)**

a) $f(-2) = 2(-2)^3 + 3(-2)^2 - 29(-2) - 60$

$= -6$

b/ $f(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$

$= 0.$

$\therefore$ $(x+3)$ is a factor.

c/

$$\begin{array}{r|l} & 2x^2 - 3x - 20 \\ \hline x+3 & 2x^3 + 3x^2 - 29x - 60 \\ & 2x^3 + 6x^2 \\ & \quad -3x^2 - 29x \\ & \quad -3x^2 - 9x \\ & \qquad -20x - 60 \\ & \qquad -20x - 60 \\ \hline & \qquad\quad 0 \end{array}$$

$(x+3)(2x^2 - 3x - 20)$

$(x+3)(2x+5)(x-4)$

5. $$f(x) = x^3 + 4x^2 + x - 6.$$

(*a*) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(*b*) Factorise $f(x)$ completely.

(4)

(*c*) Write down all the solutions to the equation

$$x^3 + 4x^2 + x - 6 = 0.$$

(1)

a) $f(-2) = (-2)^3 + 4(-2)^2 + (-2) - 6$

$= 0$

$\therefore$ $(x+2)$ is a factor.

b/

$$\begin{array}{r|l} & x^2 + 2x - 3 \\ \hline x+2 & x^3 + 4x^2 + x - 6 \\ & x^3 + 2x^2 \\ & \quad 2x^2 + x \\ & \quad 2x^2 + 4x \\ & \qquad -3x - 6 \\ & \qquad -3x - 6 \\ & \qquad\quad 0 \end{array}$$

$(x+2)(x^2+2x-3)$

$\underline{(x+2)(x+3)(x-1)}$

c/

$\underline{x = -2}$ $\underline{x = -3}$ $\underline{x = 1}$

6. (a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$,

(ii) $x + 2$.

(3)

(*b*) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0.$$

(4)

a/ i) $f(3) = (3)^3 - 2(3)^2 - 4(3) + 8$

$= 5$

ii/ $f(-2) = (-2)^3 - 2(-2)^2 - 4(-2) + 8$

$= 0$

b/

$$\begin{array}{r|l} & x^2 - 4x + 4 \\ \hline x+2 & x^3 - 2x^2 - 4x + 8 \\ & x^3 + 2x^2 \\ & \quad -4x^2 - 4x \\ & \quad -4x^2 - 8x \\ & \qquad 4x + 8 \\ & \qquad 4x + 8 \\ \hline & \qquad 0 \end{array}$$

$(x+2)(x^2 - 4x + 4)$

$(x+2)(x-2)(x-2)$

$x = -2 \quad x = 2$

7. $$f(x) = 2x^3 - 3x^2 - 39x + 20$$

(*a*) Use the factor theorem to show that $(x + 4)$ is a factor of $f(x)$. **(2)**

(*b*) Factorise $f(x)$ completely. **(4)**

a) $f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20$

$= 0$

$\therefore (x+4)$ is a factor

$$\begin{array}{r} 2x^2 - 11x + 5 \\ x+4 \overline{)\,2x^3 - 3x^2 - 39x + 20} \\ 2x^3 + 8x^2 \\ -11x^2 - 39x \\ -11x^2 - 44x \\ 5x + 20 \\ 5x + 20 \\ \hline 0 \end{array}$$

$(x+4)(2x^2 - 11x + 5)$

$\underline{(x+4)(2x-1)(x-5)}$

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8. $$f(x) = (3x - 2)(x - k) - 8$$

where k is a constant.

(*a*) Write down the value of $f(k)$. **(1)**

When $f(x)$ is divided by $(x - 2)$ the remainder is 4.

(*b*) Find the value of k. **(2)**

(*c*) Factorise $f(x)$ completely. **(3)**

a/ $f(k) = (3k-2)(k-k) - 8$

$= 0 - 8$

$\underline{= -8}$

b/ $f(2) = 4$

$(3(2)-2)(2-k) - 8 = 4$

$4(2-k) = 12$

$2-k = 3$

$k = -1$

c/ $(3x-2)(x+1) - 8$

$3x^2 + 3x - 2x - 2 - 8$

$3x^2 + x - 10 \qquad 7$

$\underline{(3x-5)(x+2)}$

9. $$f(x) = x^4 + 5x^3 + ax + b,$$

where a and b are constants.

The remainder when f(x) is divided by $(x - 2)$ is equal to the remainder when f(x) is divided by $(x + 1)$.

(a) Find the value of a.

(5)

Given that $(x + 3)$ is a factor of f(x),

(b) find the value of b.

(3)

a/ $f(2) = f(-1)$

$(2)^4 + 5(2)^3 + 2a + b = (-1)^4 + 5(-1)^3 - a + b$

$56 + 2a \cancel{+b} = -4 - a \cancel{+b}$

$3a = -60$

$a = -20$

b/ $f(-3) = 0$

$(-3)^4 + 5(-3)^3 - 20(-3) + b = 0$

$6 + b = 0$

$b = -6$