1) Assume that x and y are integers and

$$6x + 9y = 1$$

$$3(2x + 3y) = 1$$

$$2x + 3y = \frac{1}{3}$$

if x and y are integers $2x$ and $3y$ must be integers.

Integer + Integer cannot = $\frac{1}{3}$

$\therefore$ assumption must be false, no integers ~~x and y~~ exist for which $6x + 9y = 1$

2) Assume x and y are integers and

$$30x + 20y = 7$$

$$10(3x + 2y) = 7$$

$$3x + 2y = \frac{7}{10}$$

$3x$ and $2y$ must be integers.
Integer + Integer = Integer $\therefore$ assumption must be incorrect.
$\therefore$ No integers exist for x and y for which $30x + 20y = 7$.

3/ assume $\sqrt{3}$ can be expressed in the form $\frac{a}{b}$ where a and b are integers with no common factor (except 1)

$$\sqrt{3} = \frac{a}{b}$$

$$3 = \frac{a^2}{b^2}$$

$$3b^2 = a^2$$

a^2 (and a) must be multiples of 3

$$3b^2 = (3n)^2$$
$$3b^2 = 9n^2$$
$$\cancel{3}b^2 = 3n^2$$

b^2 (and b) must also be multiples of 3.

a and b have common factor 3. $\therefore$ the assumption is incorrect and $\sqrt{3}$ is irrational

4/ Assume $\sqrt{2}$ can be expressed in the form $\frac{a}{b}$ where a and b are integers with no common factors except 1.

$$\sqrt{2} = \frac{a}{b}$$

$$2 = \frac{a^2}{b^2}$$

$$2b^2 = a^2$$

a^2 (and a) must be even

$$2b^2 = (2n)^2$$
$$2b^2 = 4n^2$$

$$b^2 = 2n^2$$

b^2 (and b) must also be even

a and b have a common factor of 2. $\therefore$ the assumption is incorrect and $\sqrt{2}$ is irrational

5/ Assume the sum of a rational number and an irrational number is rational

Let the rational number $= \frac{a}{b}$ (where a and b are integers)

Let the irrational number $= c$ (which cannot be expressed in the form $\frac{a}{b}$ where a and b are integers)

Let their sum $= \frac{d}{e}$ (where d and e are integers)

$$\frac{a}{b} + c = \frac{d}{e}$$

$$c = \frac{d}{e} - \frac{a}{b}$$

$$= \frac{bd}{be} - \frac{ae}{be}$$

$$= \frac{bd - ae}{be}$$

c has been written in the form $\frac{a}{b}$ where a and b are integers. $\therefore$ the assumption is incorrect and the sum of a rational and irrational number must be irrational.

6) assume there are a finite number of primes.

Let x = the product of all prime numbers
$= P_1 \times P_2 \times P_3 \times \ldots \times P_n$

Let $y = x + 1$

y has no prime factors P_1 to P_n as there would always be a remainder of 1.

$\therefore$ either y is prime and the assumption is incorrect

or y is not prime and has a factor not listed and the assumption is incorrect.

Either way the assumption is incorrect and there are an infinite number of primes.