| Surname | Other Names |
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## Mathematics

## Practice Set A

## Paper 3 (Calculator)

 Higher Tier
## Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided
- there may be more space than you need.
- Calculators may be used.
- Diagrams are NOT accurately drawn, unless otherwise indicated.

- You must show all your working out.


## Information

- The total mark for this paper is 80
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

1 The average daytime temperature for 10 days is recorded.
A shop also records its ice cream sales for each of the 10 days.
The scatter graph shows this information.

(a) What type of correlation does the scatter graph show?
(b) On the $11^{\text {th }}$ day the temperature was $12^{\circ}$.

Estimate the ice cream sales on the 11th day.

$$
\begin{equation*}
\pm 140 \tag{2}
\end{equation*}
$$

(c) The shop's manager wants to use the scatter graph to predict the ice cream sales for a day with an average temperature of $2^{\circ}$. Comment on the reliability of this prediction.



2 Solve

$$
\begin{aligned}
& 8 t-19=5 t-11 \\
& -5 t \quad-5 t \\
& 3 t-19=-11 \\
& +19 \quad+19 \\
& 3 t=8 \\
& t=\frac{8}{3}
\end{aligned}
$$

$$
t=\frac{8}{3}
$$

(Total for Question 2 is 2 marks)
3 Bob is going to make some orange paint.
He needs to mix red paint, yellow paint and white paint in the ratio $7: 6: 2$
Bob wants to make $750 \mathrm{~m} l$ of orange paint.

$$
7+6+2=15
$$

Bob has

$$
\begin{aligned}
& 400 \mathrm{~m} l \text { of red paint } \\
& 300 \mathrm{~m} l \text { of yellow paint } \\
& 200 \mathrm{~m} l \text { of white paint }
\end{aligned}
$$

Does Bob have enough red paint, yellow paint and white paint to make the orange paint?
You must show all your working.

$$
\begin{aligned}
& 750 \div 15=50 \\
& 7 \times 50=350 \mathrm{ml} \text { of red paint } \\
& 6 \times 50=300 \mathrm{ml} \text { of yellow paint } \\
& 2 \times 50=100 \mathrm{ml} \text { ot white paint }
\end{aligned}
$$

Bob has the paint he needs

4 Hannah is going to play one game of chess and one game of backgammon.
The probability she will win the game of chess is 0.7
The probability she will win the game of backgammon is 0.8
(a) Complete the probability tree diagram.

## Chess

Backgammon

(b) Work out the probability that Hannah will win both games.

$$
0.7 \times 0.8
$$

5


Calculate the size of angle $A C B$.

$$
\begin{aligned}
\sin x & =\frac{0}{H} \\
\sin x & =\frac{7}{18} \\
x & =\sin ^{-1}\left(\frac{7}{18}\right) \\
& =22.885 \ldots
\end{aligned}
$$

22.9.
(Total for Question 5 is 2 marks)

6
(a) Factorise fully $18 a^{2} b+12 a b^{2}$
(b) Expand and Simplify $5(2 y-5)-3(2 y-1)$
(2)

$$
10 y-25-6 y+3
$$

$$
4 y-22
$$

7 The diagram shows a trapezium with an area of $45 \mathrm{~cm}^{2}$ and a perpendicular height $h \mathrm{~cm}$.


Find the value of $h$.

$$
\begin{align*}
\frac{1}{2}(6+9) \times h & =45 \\
7.5 h & =45 \\
h & =\frac{45}{75} \\
& =6 \tag{6}
\end{align*}
$$

$$
h=
$$

8 (a) Write $7.329 \times 10^{6}$ as an ordinary number.
(b) Write 0.0508 in standard form.
$5.08 \times 10^{-2}$
(1)
(c) Calculate $\left(5.51 \times 10^{4}\right) \div\left(5.8 \times 10^{-3}\right)$

Give your answer in standard form.

$$
9500000
$$

$$
\begin{equation*}
9.5 \times 10^{6} \tag{2}
\end{equation*}
$$



The diagram shows a regular pentagon, ABCDE , and a square, EDFG.
The lines $C D$ and $D G$ are both sides of another regular polygon, $P$.
How many sides does polygon P have?
You must show how you got your answer.
Exterior angle of pentagon $=\frac{360}{5}=72$

Interior angle of $p=90+72$

$$
=1 \in 2
$$

Exterior angle of $p=180-162$

$$
=18^{\circ}
$$

$$
\frac{360}{18}=20
$$



The two triangles $A B C$ and $P Q R$ are mathematically similar.
Angle $A=$ angle $P$
Angle $B=$ angle $Q$
$A B=9 \mathrm{~cm}$
$A C=21 \mathrm{~cm}$
$P Q=12 \mathrm{~cm}$
$\underset{\text { factor }}{\operatorname{scale}}=\frac{12}{9}=\frac{4}{3}$
$Q R=36 \mathrm{~cm}$
(a) Calculate the length of $P R$.

$$
21 \times \frac{4}{3}
$$

(b) Calculate the length of $B C$.

$$
36 \div \frac{4}{3}
$$

27

11 A population of bacteria is increasing by $12 \%$ each hour.
Find the percentage increase in the population every 3 hours.

$$
\begin{aligned}
& 100 \times 1.12^{3}=140.4928 \\
& \text { Increase of } 40.4928 \%
\end{aligned}
$$

$$
40.5 \%
$$

12 Light A flashes every 5 seconds.
Light B flashes every 6 seconds.
Light $\mathbf{C}$ flashes every 7 seconds.
All three lights flash at the same time.
How many times in one hour will the three lamps flash at the same time?

$$
\begin{aligned}
L C M & =5 \times 6 \times 7 \\
& =210
\end{aligned}
$$

They flash at the same time every 210

$$
\text { In one hour there are } 60 \times 60=3600 \text { seconds }
$$

$$
\frac{3600}{210}=17.14
$$

$\qquad$ Either 17 or 18 times (Total for Question 12 is 3 marks)

13 Bernie is filling up a swimming pool.
The graph shows the volume $v$ of water in the swimming pool at time $t$ hours.


Work out the rate of that the water is flowing into the pool.
Give your answer in litres per hour.

$$
\frac{37600-8000}{10}=2960 \mathrm{l} / \mathrm{hr}
$$

$2960 \ldots .1 / \mathrm{hr}$
(Total for Question 13 is 2 marks)

$$
[2900-3000]
$$

14 Cylinder A and cylinder B are mathematically similar.
The surface area of cylinder A is $50 \mathrm{~cm}^{2}$ and thetength of cylinder B is $128 \mathrm{~cm}^{2}$
The volume of cylinder $A$ is $80 \mathrm{~cm}^{3}$
Calculate the volume of cylinder B.
scale factor for area $=\frac{128}{50}=\frac{64}{25}$
scale factor for length $=\frac{8}{5}$

Scale factor for volume $=\frac{512}{125}$

$$
80 \times \frac{512}{125}=327.68 \mathrm{~cm}^{3}
$$

15 There are 12 people in a room.
Each person shakes each other person's hand once.
Work out the number handshakes that take place.

$$
\frac{12 \times 11}{2}=66
$$

16 By completing the square, find the coordinates of the turning point of the curve with the equation $y=x^{2}-3 x+1$
You must show all your working.

$$
\begin{aligned}
& y=\left(x-\frac{3}{2}\right)^{2}-\frac{9}{4}+1 \\
& y=\left(x-\frac{3}{2}\right)^{2}-\frac{5}{4}
\end{aligned}
$$

$$
\left(\frac{3}{2}-\frac{5}{4}\right)
$$

17 Here are the first 5 terms of a quadratic sequence.

$$
6
$$

Find an expression, in terms of $n$, for the $n$th term of this sequence.

$$
\begin{array}{ll}
a+b \times c \rightarrow 6 \quad 10 \quad 17 \\
3 a+b & 4 n^{2}+n^{n}+1 \\
2 a \rightarrow 3
\end{array}
$$

$$
\begin{array}{rlrl}
2 a=3 & 3 a+b & =4 & a+b+c
\end{array}=6
$$

$$
1.5 n^{2}-0.5 n+5
$$

18 Given that

$$
x+9: 5 x-1=x+7: 2 x-3
$$

Find the possible values of $x$.

$$
\begin{array}{rl}
\frac{x+9}{5 x-1} & =\frac{x+7}{2 x-3} \\
(x+9)(2 x-3) & =(x+7)(5 x-1) \\
2 x^{2}-3 x+18 x-27 & =5 x^{2}-x+35 x-7 \\
2 x^{2}+15 x-27 & =5 x^{2}+34 x-7 \\
0 & =3 x^{2}+19 x+20 \\
0 & =(3 x+4)(x+5) \\
x & x-\frac{4}{3} \quad x=-5
\end{array}
$$

$$
x=\frac{-4}{3} \text { or }-5
$$

(Total for Question 18 is 4 marks)

19


Angle $B A C$ is acute and the area of the triangle is $45 \mathrm{~m}^{2}$ Calculate the perimeter of triangle $A B C$.
Give your answer to 3 significant figures.

$$
\begin{aligned}
& \frac{1}{2} a b \sin c=45 \\
& \frac{1}{2}(6.7)(14.8) \sin x=45 \\
& 49.58 \sin x=45 \\
& \sin x=\frac{45}{4958} \\
& x=\sin ^{-1}\left(\frac{45}{49.58}\right) \\
& =\underline{ } \\
& B C^{2}=6.7^{2}+14.8^{2}-2(6: 7)(14.8) \cos (65.179) \\
& =180.678 \\
& B C=\sqrt{180.678} \\
& =13.4 \mathrm{~m} 3 \mathrm{t} \\
& \text { perimeter }=6.7+14.8+13.4 \\
& =34.9 \mathrm{~m}
\end{aligned}
$$

20 $A B C D$ is a parallelogram


Prove that angle $A B C$ is and triangle $B C D$. are congruent.

$$
\begin{aligned}
& A B=C D \text { opposite sides of a parallelogram } \\
& \text { are equal }
\end{aligned}
$$

$$
A C=B D
$$

$B C$ is common to both triangles
SSS

21 (a) Show that the equation $x^{3}+4 x=1$ has a solution between $x=0$ and $x=1$.

$$
\begin{aligned}
& (0)^{3}+4(0)=0 \\
& (1)^{3}+4(1)=5
\end{aligned}
$$

$$
1 \text { is between } 0 \text { and } 5 \text { (and the }
$$

function is continuous) $\therefore$ solution between O and 1
(b) Show that the equation $x^{3}+4 x=1$ can be rearranged to give: $x=\frac{1}{4}-\frac{x^{3}}{4}$

$$
\begin{aligned}
x^{3}+4 x & =1 \\
4 x & =1-x^{3} \\
x & =\frac{1}{4}-\frac{x^{3}}{4}
\end{aligned}
$$

(c) Starting with $x_{0}=0$, use the iteration formula $\quad x_{n+1}=\frac{1}{4}-\frac{x_{n}^{3}}{4}$ twice to find an estimate for the
solution to $x^{3}+4 x=1$

$$
\begin{aligned}
& x_{1}=\frac{1}{4}-\frac{(0)^{3}}{4}=\frac{1}{4} \\
& x_{2}=\frac{1}{4}-\frac{A^{3} s^{3}}{4}=\frac{63}{256}=0.24609375
\end{aligned}
$$

$\qquad$

2250 people were asked which fruits they liked from apples, bananas and oranges.
12 people like all three fruits.
34 people like apples.
7 like apples and bananas but not oranges.
16 like bananas and oranges.
4 of the people do not like any of the fruits.
All 25 people who like oranges like at least one other fruit.
Two of the 50 people are chosen at random.
Work out the probability that they both like bananas.


$$
=\frac{4}{175}
$$

23


Angles at the base of an isosceles triangle are equal
$A, B$ and $C$ are points on the circumference of a circle, centre $O$. $A O C$ is a diameter of the circle.

Prove that angle $A B C$ is $90^{\circ}$
You must not use any circle theorems in your proof.

Let $A B O=x \quad$ Let OBC=y

$$
\frac{A B C=x+y}{A O B=180-2 x \quad B O C=180-2 y}
$$

$$
\begin{aligned}
& A O B+B O C=180 \quad \text { Angles on a straight } \\
& \text { line add to } 180^{\circ} \\
& 180-2 x+180-2 y=180 \\
& 360-2 x-2 y=180 \\
& 360=180+2 x+2 y \\
& 180=2 x+2 y \\
& A B C=x+y \therefore A B C
\end{aligned}
$$

