Mathematics
November 2022 Practice Paper 3 (Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
– there may be more space than you need.
- **Calculators may be used**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2}(a + b)\, h$$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

$$\text{Circumference of a circle} = 2\pi r = \pi d$$

$$\text{Area of a circle} = \pi r^2$$
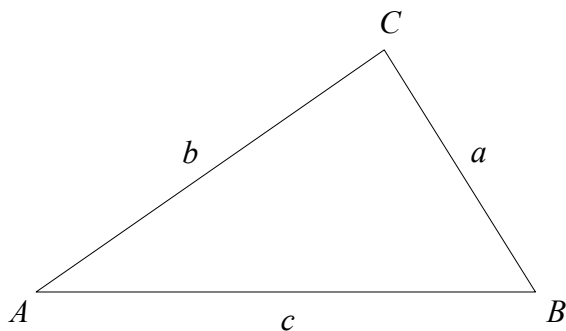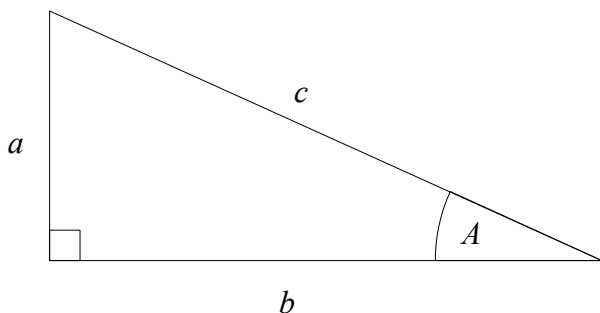
Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \dfrac{1}{2} ab \sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

$$P (A \text{ or } B) = P (A) + P (B) - P (A \text{ and } B)$$

$$P (A \text{ and } B) = P (A \text{ given } B)\, P(B)$$

END OF EXAM AID

1 $m = n - 5p$

Make p the subject of the formula.

$$M + 5p = n$$
$$5p = n - M$$
$$p = \frac{n - M}{5}$$

$$p = \frac{n - M}{5}$$

(Total for Question 1 is 2 marks)

2   (a) Write 5.2×10^{-1} as an ordinary number.

$$0.52$$

(1)

(b) Work out the value of $(3.2 \times 10^3) \times (6.5 \times 10^4)$
Give your answer in standard form.

$$208000000$$

$$2.08 \times 10^8$$

(2)

(Total for Question 2 is 3 marks)

3 Write 30 kilometres per hour in metres per second

$30\,000$ metres per hour

$\div 60$

500 metres per minute

$\div 60$

$\dfrac{25}{3}$ m/s

.... $\dfrac{25}{3}$ .... m/s

(Total for Question 3 is 2 mark)

4 In a bag there are blue sweets, red sweets and yellow sweets.

The number of red sweets is three times the number of blue sweets.
The number of yellow sweets is half the number of red sweets.

Write down the ratio of blue sweets to red sweets to yellow sweets.
Give your answer in the form $a : b : c$ where a, b and c are whole numbers

B : R : Y

1 : 3 : 1.5

2 : 6 : 3

.... 2 : 6 : 3

(Total for Question 4 is 2 marks)

5 Bob is going to make some orange paint.
He needs to mix red paint, yellow paint and white paint in the ratio 5 : 4 : 1

10 parts

Bob wants to make 750 m*l* of orange paint.

Bob has
  400 m*l* of red paint
  300 m*l* of yellow paint
  200 m*l* of white paint

Does Bob have enough red paint, yellow paint and white paint to make the orange paint?
You must show all your working.

$$\frac{750}{10} = 75$$

Red: 5 × 75 = 375 ml needed ✓
Yellow: 4 × 75 = 300 ml needed ✓
White : 75 ml needed ✓

Yes

(Total for Question 5 is 4 marks)

6 A shop sells small chocolate bars and large chocolate bars.

There are
  small chocolate bars are sold in packs of 4
  large chocolate bars are sold in packs of 9

On one day

$$\frac{\text{the number of packs of}}{\text{small chocolate bars sold}} : \frac{\text{the number of packs of}}{\text{large chocolate bars sold}} = 5 : 2$$

$$\overset{\times 4}{5} : \overset{\times 9}{2}$$

A total of 266 chocolate bars were sold.

$$20 : 18 \quad \text{Bars Sold}$$

Work out the number of small chocolate bars sold.
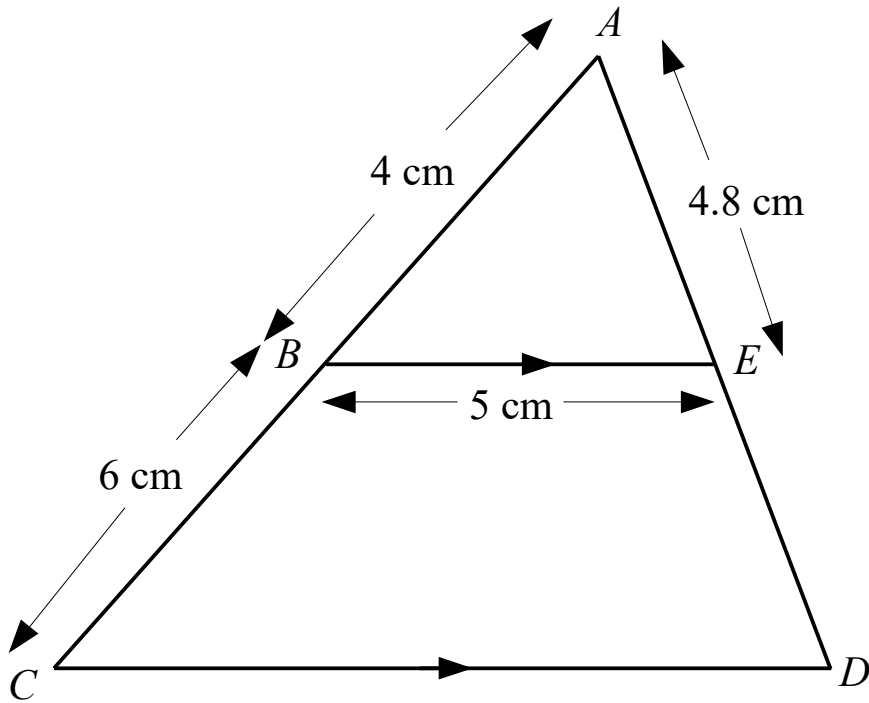
$$20 + 18 = 38$$

$$\frac{266}{38} = 7$$

$$20 \times 7 \quad : \quad 18 \times 7$$
$$\underline{\underline{140}} \quad : \quad 126$$
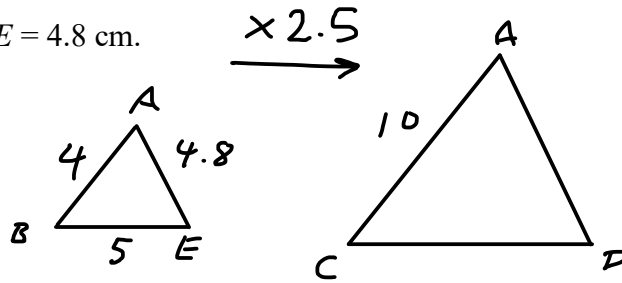
140

(Total for Question 6 is 4 marks)

7

BE is parallel to *CD*.
ABC and *AED* are straight lines.
AB = 4 cm, *BC* = 6 cm, *BE* = 5 cm, *AE* = 4.8 cm.

×2.5 →

(a)   Calculate the length of *CD*.

$\frac{10}{4} = 2.5$

$5 \times 2.5 = 12.5$

12.5 _____ cm

(2)

(b)   Calculate the length of *ED*.

$AD = 4.8 \times 2.5$
$= 12$

$ED = 12 - 4.8$
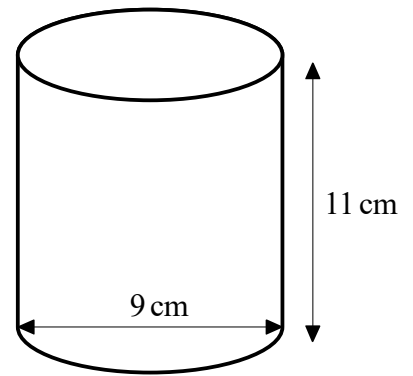$= 7.2$

7.2 _____ cm

(2)

(Total for Question 7 is 4 marks)

8 A cylinder has a diameter of 9 cm and a height of 11 cm.

$r = 4.5$

Work out the volume of the cylinder.
Give your answer correct to 1 decimal place.

11 cm

9 cm

$$V = \pi r^2 h$$

$$= \pi (4.5)^2 (11)$$

$$= 699.7897636$$

$\underline{699.8}$ cm³

(2)

(b) The volume of another cylinder is 1500 cm³.
Michael says that 1500cm³ is the same as 15 m³.

Is Michael correct?
You must give a reason for your answer.

$1 m^3 = 100 cm \times 100 cm \times 100 cm$

$= 1000000 \ cm^3$

No. $1 m^3 = 1000000 \ cm^3$

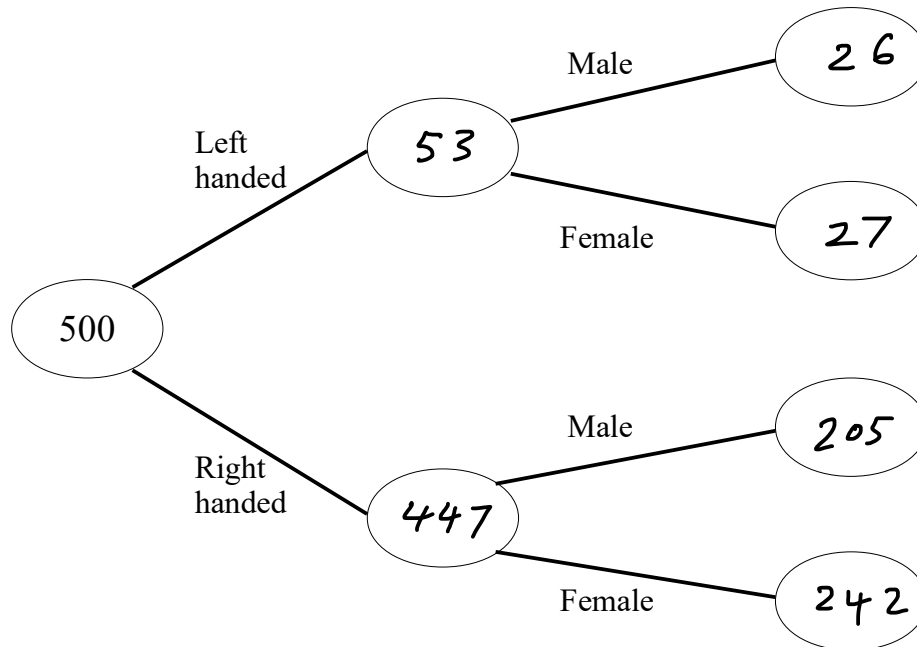$15 m^3 = 15000000 \ cm^3$

(1)

(Total for Question 8 is 3 marks)

9 500 people were surveyed.
All of the people were either left handed or right handed.

53 of the people are left handed.
26 males are left handed.
231 of the people are male.

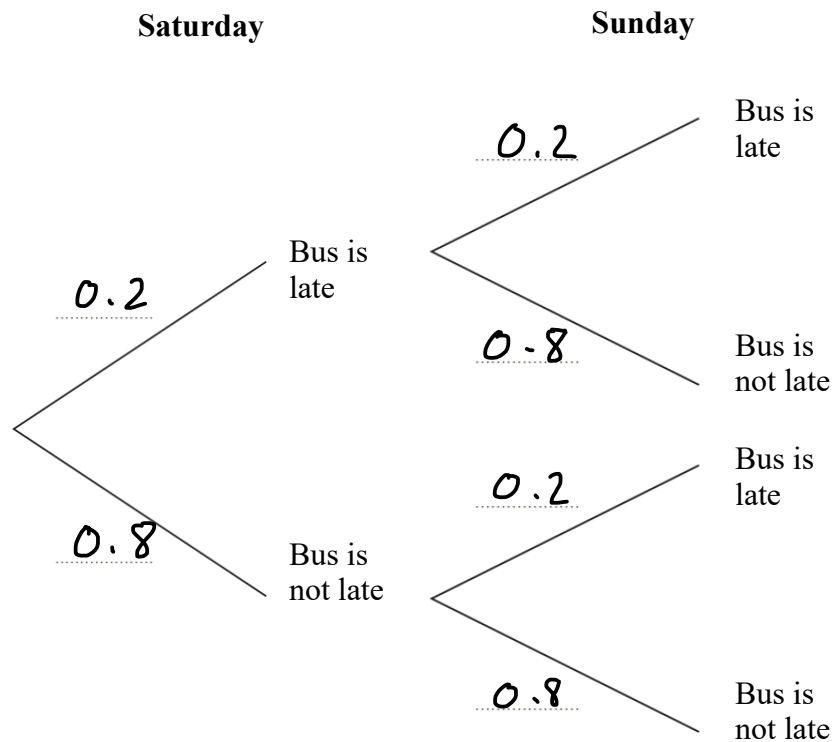(a) Use this information to complete the frequency tree.

(Total for Question 9 is 2 marks)

10 Bradley gets the bus on Saturday and Sunday.
The probability that Bradley's bus will be late on any day is 0.2

(a) Complete the probability tree diagram.

Saturday   Sunday

0.2 Bus is late

0.2 Bus is late

0.8 Bus is not late

0.2 Bus is late

0.8 Bus is not late

0.8 Bus is not late

(2)

(b) Work out the probability that Bradley's bus is late on at least one of these days.

P (Not Late, Not Late) = 0.8 × 0.8
                        = 0.64

1 − 0.64 = 0.36

0.36

(2)

(Total for Question 10 is 4 marks)

11   Michael recorded the maximum temperature every day in September.

The table shows information about his results.

Temperature (°C)	M.P	Frequency	mp x f
14 < t ≤ 18	16	4	64
18 < t ≤ 20	19	10	190
20 < t ≤ 22	21	8	168
22 < t ≤ 24	23	5	115
24 < t ≤ 28	26	3	78
		30	615

Calculate an estimate for the mean maximum temperature.
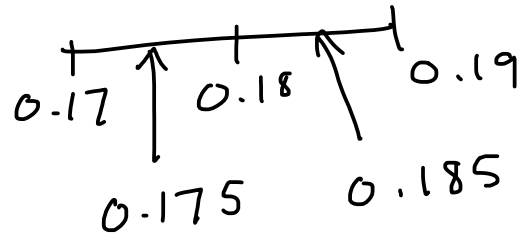
$$\frac{615}{30} = 20.5$$

....................20.5.................... °C

(Total for Question 11 is 3 marks)

12 A number x is rounded to 2 decimal places.

The result is 0.18

Write down the error interval for x.

0.17   0.18   0.19

0.175   0.185

$0.175 \leq x < 0.185$

(Total for Question 12 is 2 marks)

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13 There are 30 students in a class.
Two students are going to be selected to receive a prize.

How many different pairs of students could be selected?

$$\frac{30 \times 29}{2} = 435$$

435

(Total for Question 13 is 2 marks)

14 $y^2 \times y^a = y^7$

(a) Find the value of a.

..................**5**..................

(1)

$(y^4)^b = y^{12}$

(b) Find the value of b.

..................**3**..................

(1)

(Total for Question 14 is 2 marks)

15 Expand and Simplify $(2x + 1)(x + 2)(x + 3)$

$$(2x^2 + 4x + x + 2)(x + 3)$$

$$(2x^2 + 5x + 2)(x + 3)$$

$$2x^3 + 6x^2 + 5x^2 + 15x + 2x + 6$$

$$2x^3 + 11x^2 + 17x + 6$$

..................$2x^3 + 11x^2 + 17x + 6$..................

(Total for Question 15 is 3 marks)

16 100ml of liquid A and 200ml of liquid B are mixed together to make liquid C.
Liquid A has a density of 0.7g/ml.
Liquid B has a density of 1.1 g/ml.

Work the density of liquid C.
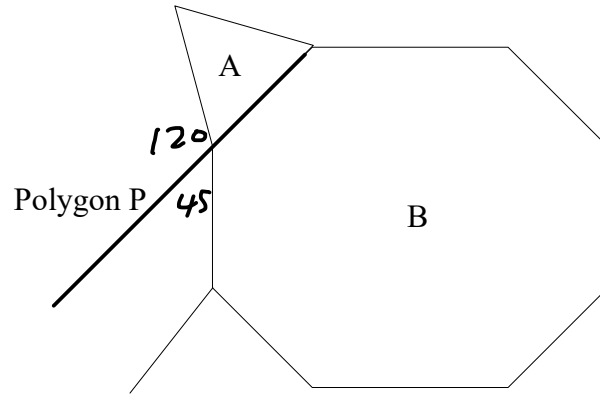
$$d = \frac{M}{V}$$

$$M = d \times V$$

A/ $M = 0.7 \times 100$

$= 70g$

B/ $M = 1.1 \times 200$

$= 220g$

C/ $d = \dfrac{total\ M}{total\ v}$

$= \dfrac{220 + 70}{100 + 200}$

$= \dfrac{29}{30}$

.... $0.9\dot{6}$ .... g/ml

(Total for Question 16 is 4 marks)

17

A

120

Polygon P 45

B

Shape A is a regular triangle. Shape B is a regular octagon.

Another regular polygon, P, is shown on the diagram.

How many sides does polygon P have?

You must show your working.

$$\frac{360}{8} = 45$$

(ext. angle of octagon)

$$\frac{360}{3} = 120$$

(ext. angle of triangle)

Int angle of P = 45 + 120
= 165°

Ext. angle of P = 180 − 165
= 15°

$$\frac{360}{15} = \underline{\underline{24}}$$

.........24.........

(Total for Question 17 is 4 marks)

18
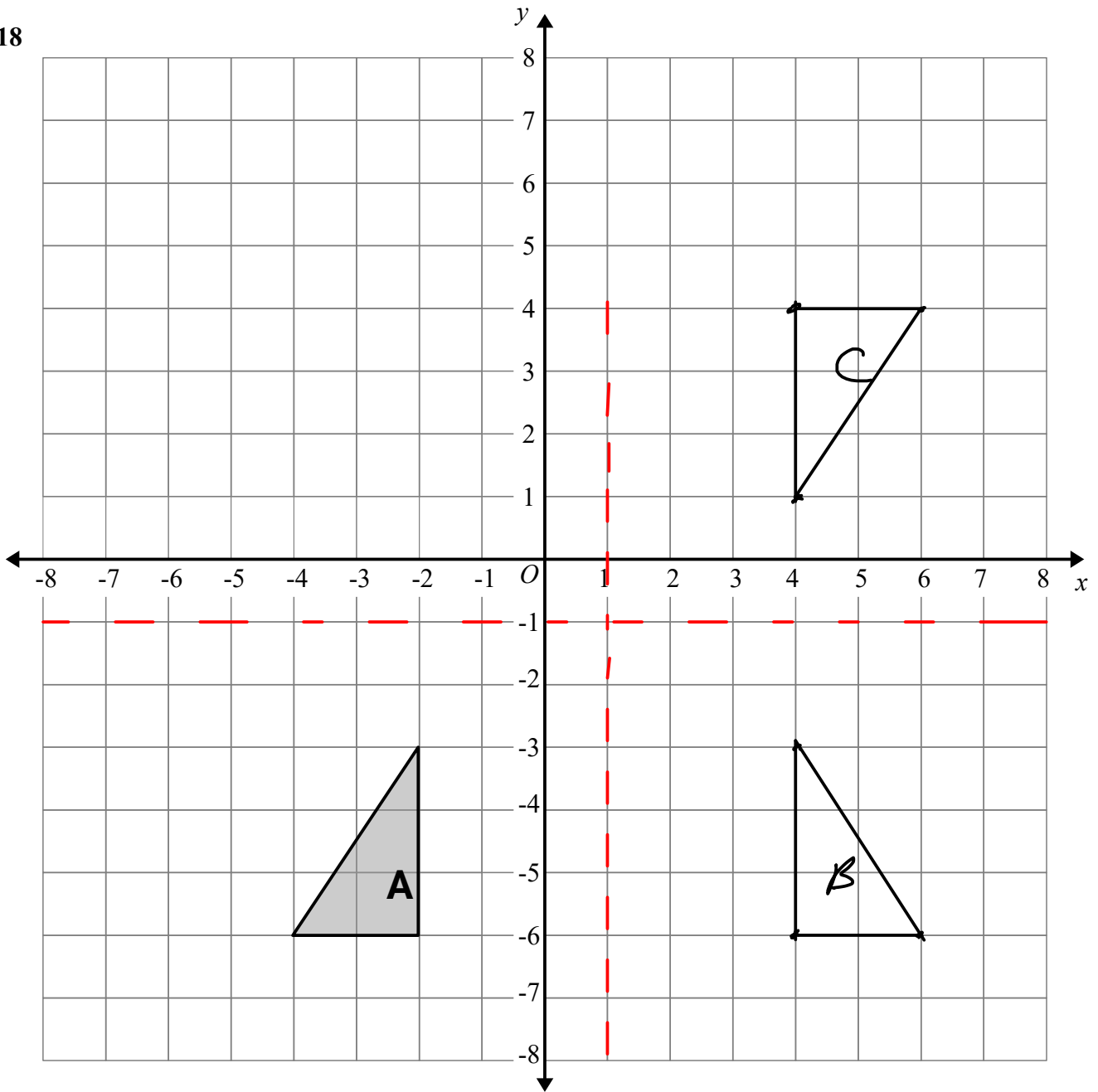
Shape **A** is reflected in the line $x = 1$ to give shape **B**.
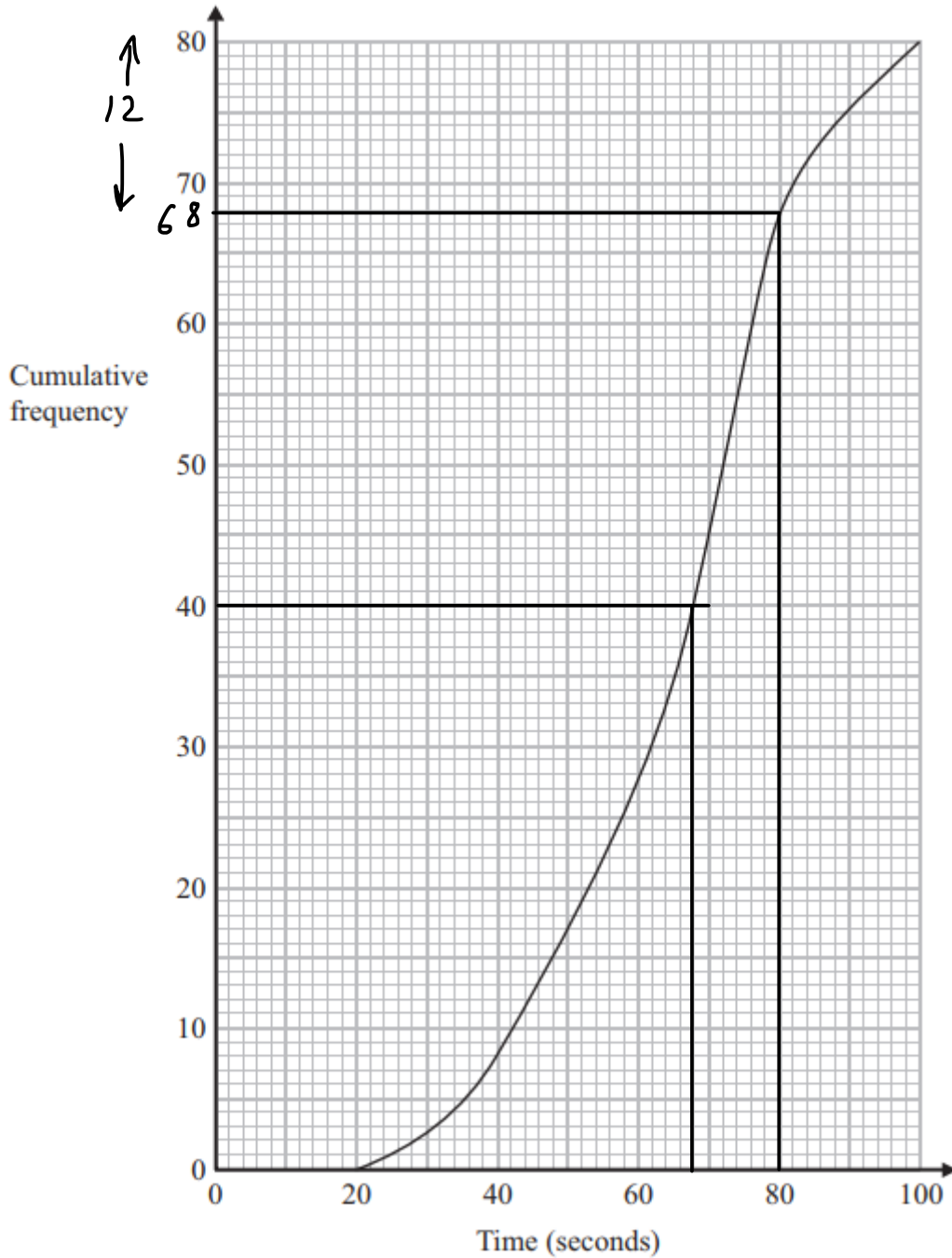Shape **B** is reflected in the line $y = -1$ to give shape **C**.

Describe the **single** transformation that will map shape **A** to shape **C**.

Rotation, 180°, centre (1, -1)

....

....

(Total for Question 18 is 2 marks)

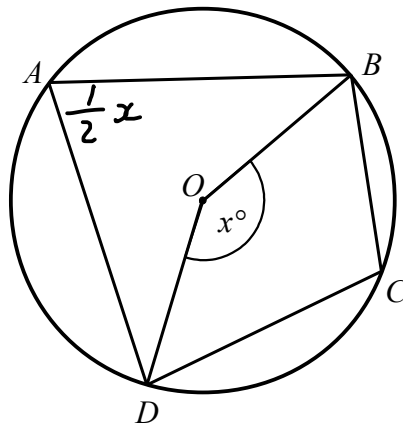**19** The cumulative frequency graph gives some information the times it took people to complete a challenge.

(a) Find the median time.

..............**68**..............seconds
(1)

(b) Find the number of people who took longer then 80 seconds to complete the challenge.

..............**12**..............
(1)

(Total for Question 19 is 2 marks)

20

A, B, C and D are points on the circumference of a circle, centre O.

Angle $BOD = x°$

Find the size of angle BCD, in terms of x.
Give reasons for each stage of your working.

$BAD = \frac{1}{2}x$ Angle at the circumference is half the angle at the centre

$BCD = 180 - \frac{1}{2}x$ opposite angles in a cyclic quadrilateral add to 180°

(Total for Question 20 is 3 marks)

21 Simplify fully $\dfrac{3x+6}{x-4} \div \dfrac{2x^2+9x+10}{x^2-4x}$

$$\frac{3(x+2)}{x-4} \times \frac{x^2-4x}{2x^2+9x+10}$$

$$\frac{3(x+2)}{(x-4)} \times \frac{x(x-4)}{(2x+5)(x+2)}$$

$$\frac{3x(x+2)(x-4)}{(x-4)(2x+5)(x+2)}$$

$$\frac{3x}{2x+5}$$

$$\frac{3x}{2x+5}$$

....

(Total for Question 21 is 3 marks)

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22 (a) Write $x^2 + 10x + 2$ in the form $(x+a)^2 + b$ where a and b are integers.
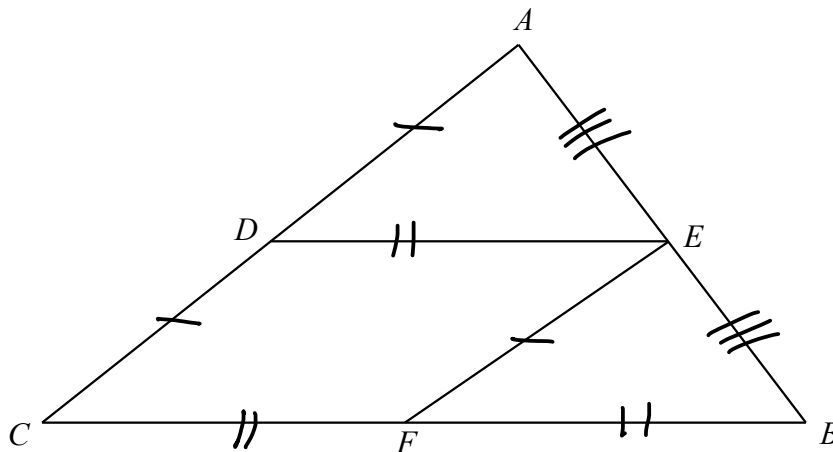
$$(x+5)^2 - 25 + 2$$

$$(x+5)^2 - 23$$

....

(2)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = x^2 + 10x + 2$

$$(-5, -23)$$

....

(1)

(Total for Question 22 is 3 marks)

23   *ABC* is a triangle.

CDEF is a parallelogram such that:
D is the midpoint of *AC*
E is the midpoint of *AB*
F is the midpoint of *BC*

Prove that triangle *ADE* is congruent to triangle *BEF*.

$AD = CD$ (D is Midpoint)

$CD = EF$ (opp sides in parallelogram are equal)

$\therefore\ AD = EF$

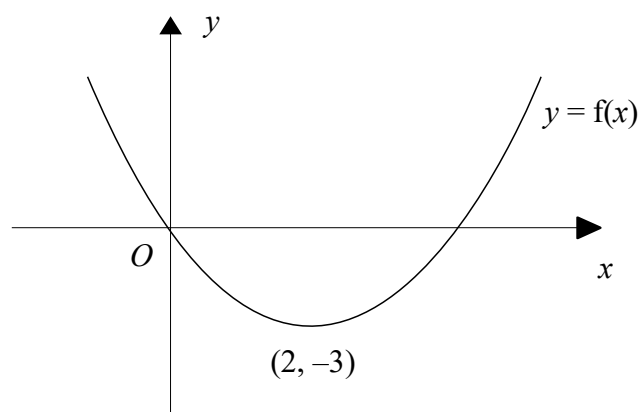$CF = BF$ (F is midpoint)

$DE = CF$ (opp sides in parallelogram are equal)

$\therefore\ BF = DE$

$AE = EB$ E is a midpoint

$\underline{\underline{SSS}}\ \therefore\ \text{congruent}$

(Total for Question 23 is 4 marks)

24 The graph of $y = f(x)$ is shown below.

The coordinates of the minimum point of this curve are (2, –3).

Write down the coordinates of the turning point of the curve with equation

(a) $y = f(x + 2)$

$(0, -3)$

(1)

(b) $y = -f(x)$

$(2, 3)$

(1)

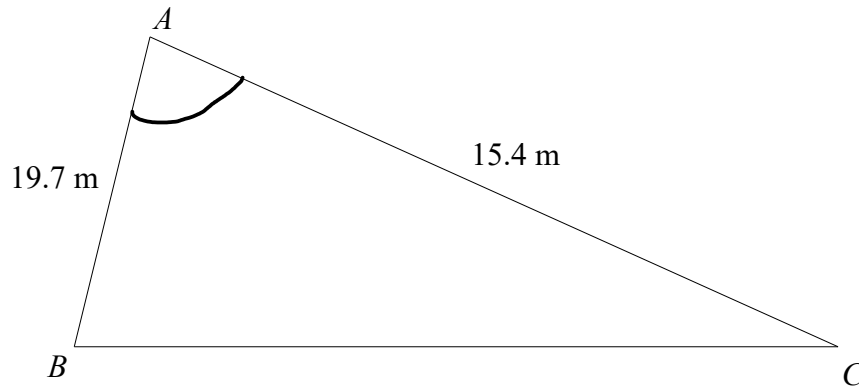(c) $y = f(x) + 2$

$(2, -1)$

(1)

(d) $y = f(-x)$

$(-2, -3)$

(1)

(Total for Question 24 is 4 marks)

25

A

19.7 m

15.4 m

B

C

The area of the triangle is 100m^2
Calculate the perimeter of triangle *ABC*.
Give your answer to 3 significant figures.

Area $= \frac{1}{2} ab \sin C$

$$\frac{1}{2}(19.7)(15.4) \sin A = 100$$

$$\sin A = \frac{100}{\frac{1}{2}(19.7)(15.4)}$$

$$A = \sin^{-1}(0.659\ldots)$$

$$= 41.24187853$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$a^2 = 19.7^2 + 15.4^2 - 2(19.7)(15.4) \cos(41.24)$$
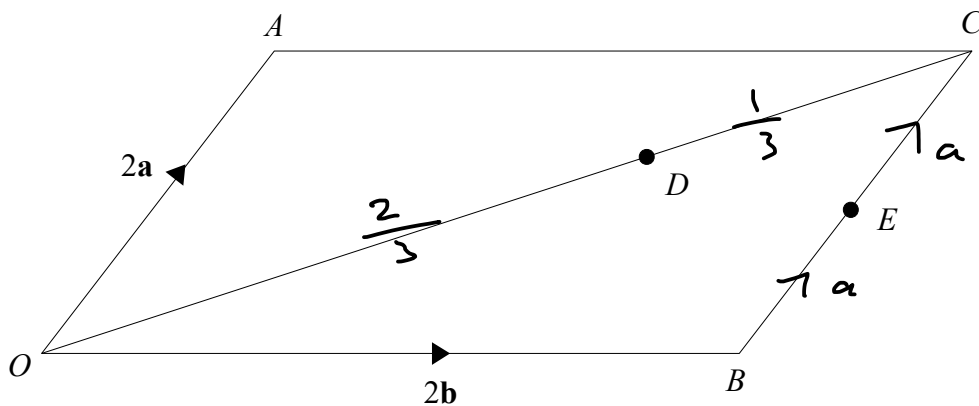
$$= 169.00697\ldots$$

$$a = \underline{13 m}$$

$$13 + 19.7 + 15.4 = 48.1 m$$

........**48.1**........m

(Total for Question 25 is 5 marks)

26   The diagram shows a parallelogram.

$\overrightarrow{OA} = 2a$

$\overrightarrow{OB} = 2b$

D is the point on OC such that OD:DC = 2:1

E is the midpoint of BC

Show that A, D and E are on the same straight line.

$$\overrightarrow{AE} = -2a + 2b + a$$

$$= \underline{2b - a}$$

$$\overrightarrow{OC} = 2b + 2a$$

$$\overrightarrow{OD} = \frac{2}{3}(2b + 2a)$$

$$= \frac{4}{3}b + \frac{4}{3}a$$

$$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$$

$$= -2a + \frac{4}{3}b + \frac{4}{3}a$$

$$= \frac{4}{3}b - \frac{2}{3}a$$

$$= \frac{2}{3}\left(2b - a\right)$$

$$\overrightarrow{AD} = \frac{2}{3}\overrightarrow{AE}$$

parallel and both
go through A ∴ same
line

(Total for Question 26 is 4 marks)