Name: ____________________________

GCSE (1 – 9)

Proof

Instructions

• Use **black** ink or ball-point pen.
• Answer all questions.
• Answer the questions in the spaces provided
  – *there may be more space than you need.*
• Diagrams are **NOT** accurately drawn, unless otherwise indicated.
• You must **show all your working out**.

Information

• The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end

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1. Prove algebraically that the sum of any two consecutive integers is always an odd number.

\[ n + n+1 \]
\[ 2n + 1 \]
\[ \text{odd} \]
\[ \text{even} + 1 \text{ is odd} \]

(Total for question 1 is 2 marks)

2. Prove algebraically that the sum of any three consecutive even integers is always a multiple of 6.

\[ 2n + 2n+2 + 2n + 4 \]
\[ 6n + 6 \]
\[ 6(n + 1) \]

(Total for question 2 is 2 marks)
3. Prove that \((3n + 1)^2 - (3n - 1)^2\) is always a multiple of 12, for all positive integer values of \(n\).

\[
\left( (3n+1)(3n+1) \right) - \left( (3n-1)(3n-1) \right) \\
(9n^2 + 6n + 1) - (9n^2 - 6n + 1) \\
9n^2 + 6n + 1 - 9n^2 + 6n - 1 \\
12n
\]

(Total for question 3 is 2 marks)

4. \(n\) is an integer.
Prove algebraically that the sum of \(n(n+1)\) and \(n+1\) is always a square number.

\[
\begin{align*}
    n(n+1) &+ n + 1 \\
    n^2 &+ n + n + 1 \\
    n^2 &+ 2n + 1 \\
    (n+1)(n+1) & \\
    (n+1)^2 &
\end{align*}
\]

(Total for question 4 is 2 marks)
5. Prove that \((2n+3)^2 - (2n-3)^2\) is always a multiple of 12, for all positive integer values of \(n\).

\[
\begin{align*}
(2n+3)(2n+3) - (2n-3)(2n-3) &= 4n^2 + 12n + 9 - (4n^2 - 12n + 9) \\
24n &= 12(2n)
\end{align*}
\]

(Total for question 5 is 2 marks)

6. \(n\) is an integer.
Prove algebraically that the sum of \((n+2)(n+1)\) and \(n+2\) is always a square number.

\[
\begin{align*}
(n+2)(n+1) + n+2 &= n^2 + n + 2n + 2 + n + 2 \\
n^2 + 4n + 4 &= (n+2)^2
\end{align*}
\]

(Total for question 6 is 2 marks)
7  Prove that the sum of 3 consecutive odd numbers is always a multiple of 3.

\[ 2n + 1 + 2n + 3 + 2n + 5 \]

\[ 6n + 9 \]

\[ 3(2n+3) \]

(Total for question 7 is 2 marks)

8  Prove that the sum of 3 consecutive even numbers is always a multiple of 6.

\[ 2n + 2n + 2 + 2n + 4 \]

\[ 6n + 6 \]

\[ 2(3n + 3) \]

\[ 6(n+1) \]

(Total for question 8 is 2 marks)
9 Prove algebraically that the sum of the squares of any 2 even positive integers is always a multiple of 4.

\[
(2n)^2 + (2m)^2 \\
4n^2 + 4m^2 \\
4(n^2 + m^2)
\]

(Total for question 9 is 2 marks)

10 Prove algebraically that the sum of the squares of any 2 odd positive integers is always even.

\[
(2n+1)^2 + (2m+1)^2 \\
(2n+1)(2n+1) + (2m+1)(2m+1) \\
4n^2 + 2n + 2n + 1 + 4m^2 + 2m + 2m + 1 \\
4n^2 + 4n + 4m^2 + 4m + 2 \\
2(2n^2 + 2n + 2m^2 + 2m + 1)
\]

(Total for question 10 is 2 marks)
11. Prove that the sum of the squares of any two consecutive integers is always an odd number.

\[
\begin{align*}
n^2 + (n+1)^2 \\
n^2 + (n+1)(n+1) \\
n^2 + n^2 + n + n + 1 \\
n^2 + n^2 + 2n + 1 \\
2n^2 + 2n + 1 \\
2(n^2 + n) + 1 \\
\text{even} \quad \text{even} + 1 \text{ is odd.}
\end{align*}
\]

(Total for question 11 is 3 marks)

12. Prove that the sum of the squares of two consecutive odd numbers is always 2 more than a multiple of 8.

\[
\begin{align*}
(2n + 1)^2 + (2n+3)^2 \\
(2n+1)(2n+1) + (2n+3)(2n+3) \\
4n^2 + 2n + 2n + 1 + 4n^2 + 6n + 6n + 9 \\
8n^2 + 16n + 10 \\
8n^2 + 16n + 8 + 2 \\
8(n^2 + 2n + 1) + 2 \\
\text{multiple of} \quad 8 + 2
\end{align*}
\]

(Total for question 12 is 2 marks)
13 Prove that the difference between the squares of any 2 consecutive integers is equal to the sum of these integers.

\[ \begin{align*}
\text{n and n+1} \\
n^2 + (n+1)^2 &= 2n+1 \\
(n+1)^2 - n^2 &= (n+1)(n+1) - n^2 \\
n^2 + n + n + 1 - n^2 &= 2n + 1 \\
\end{align*} \]

(Total for question 13 is 3 marks)

14 Prove algebraically that the sums of the squares of any 2 consecutive even number is always 4 more than a multiple of 8.

\[ \begin{align*}
(2n)^2 + (2n+2)^2 &= 4n^2 + (2n+2)(2n+2) \\
4n^2 + 4n^2 + 4n + 4n + 4 &= 8n^2 + 8n + 4 \\
8(n^2 + n) + 4 &= \underline{\text{multiple of 8}} + 4 \\
\end{align*} \]

(Total for question 14 is 3 marks)