Name: ____________________________

GCSE (1 – 9)
Probability Equation Questions

Instructions

• Use black ink or ball-point pen.
• Answer all questions.
• Answer the questions in the spaces provided
  – there may be more space than you need.
• Diagrams are NOT accurately drawn, unless otherwise indicated.
• You must show all your working out.

Information

• The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end

mathsgenie.co.uk
There are some red counters and some blue counters in a bag.

The ratio of red counters to blue counters is 3:1.

Two counters are removed at random.

The probability that both the counters taken are blue is \( \frac{1}{20} \)

Work how many counters were in the bag before any counters were removed.

\[
\begin{align*}
3x & \text{ Red} \quad x & \text{ Blue} \quad 4x & \text{ Total} \\
\frac{3x-1}{4x-1} & \text{ R} \\
\frac{3x}{4x} & \text{ R} \\
\frac{x}{4x} & \text{ B} \\
\frac{x-1}{4x-1} & \text{ B} \\
\frac{1}{20} & \\
\end{align*}
\]

\[
\frac{2}{4x} \times \frac{x-1}{4x-1} = \frac{1}{20}
\]

\[
\frac{1}{4} \times \frac{x-1}{4x-1} = \frac{1}{20}
\]

\[
\frac{x-1}{4(4x-1)} = \frac{1}{20}
\]

\[
20(x-1) = 4(4x-1)
\]

\[
20x-20 = 16x-4
\]

\[
4x-20 = -4
\]

\[
4x = 16
\]

\[
x = 4
\]

(Total for question 1 is 5 marks)
2. There are some red counters and some blue counters in a bag.

The ratio of red counters to blue counters is 4:1.

Two counters are removed at random.

The probability that both the counters taken are red is \( \frac{22}{35} \).

Work how many blue counters are in the bag.

\[
\begin{align*}
4x \text{ Red} & \quad x \text{ Blue} & 5x \text{ Total} \\
\frac{\frac{4x}{5}}{R} & \quad \frac{\frac{4x-1}{5x-1}}{R} & \frac{22}{35} \\
\frac{\frac{4x}{5}}{B} & \quad \frac{\frac{4x-1}{5x-1}}{R} & \\
\frac{\frac{4x}{5}}{B} & \quad \frac{\frac{4x-1}{5x-1}}{R} & \\
\frac{\frac{2x-1}{5x-1}}{B} & \quad \frac{\frac{2x-1}{5x-1}}{B} & \\
\end{align*}
\]

\[
\frac{\frac{4}{5} \times \frac{4x-1}{5x-1}}{R} = \frac{22}{35}
\]

\[
\frac{4(4x-1)}{5(5x-1)} = \frac{22}{35}
\]

\[
\frac{16x-4}{25x-5} = \frac{22}{35}
\]

\[
35(16x-4) = 22(25x-5)
\]

\[
560x - 140 = 550x - 110
\]

\[
10x - 140 = -110
\]

\[
10x = 30
\]

\[
x = 3
\]

(Total for question 2 is 5 marks)
3 There are 5 red counters and y blue counters in a bag.

Imogen takes a counter from the bag at random. She puts the counter back into the bag. Imogen then takes another counter at random from the bag.

The probability that the first counter Imogen takes is red and the second counter Imogen takes is red is \( \frac{1}{9} \)

Work how many blue counters are in the bag.

\[
\frac{5}{5+y} \times \frac{5}{5+y} = \frac{1}{9}
\]

\[
\frac{25}{(5+y)(5+y)} = \frac{1}{9}
\]

\[
225 = (5+y)(5+y)
\]

\[
225 = 25+5y+5y+y^2
\]

\[
225 = 25+10y+y^2
\]

\[
0 = y^2 + 10y - 200
\]

\[
0 = (y+20)(y-10)
\]

\[
y = -20 \quad y = 10
\]

\([y \text{ cannot be negative}]

(Total for question 3 is 5 marks)
There are 4 red counters and \( x \) blue counters in a bag. 2 counters are removed from the bag at random.

The probability that both the counters taken are blue is \( \frac{1}{3} \).

Work out the value of \( x \).

\[
\frac{4}{4+x} \quad \frac{3}{3+x} \quad \frac{x-1}{3+x} \quad \frac{1}{3}
\]

\[
\frac{2x}{4+x} \times \frac{x-1}{3+x} = \frac{1}{3}
\]

\[
\frac{x(x-1)}{(4+x)(3+x)} = \frac{1}{3}
\]

\[
3x(x-1) = (4+x)(3+x)
\]

\[
3x^2 - 3x = 12 + 4x + 3x + x^2
\]

\[
3x^2 - 3x = 12 + 7x + x^2
\]

\[
2x^2 - 10x - 12 = 0
\]

\[
x^2 - 5x - 6 = 0
\]

\[
(x - 6)(x + 1) = 0
\]

\[
x = 6 \quad x = -1
\]

\( x \) cannot be negative

\[
x = 6
\]

(Total for question 4 is 6 marks)
There are 5 red counters and \(x\) blue counters in a bag.

2 counters are removed from the bag at random.

The probability that both the counters taken are red is \(\frac{5}{33}\).

Work out the value of \(x\).

\[
\begin{align*}
\frac{5}{x+5} & \quad \frac{4}{x+4} \quad \frac{5}{33} \\
\frac{x}{x+4} & \quad \frac{5}{x+4} \quad \text{R} \\
\frac{2}{x+5} & \quad \frac{5}{x+4} \quad \text{B} \\
\frac{2-1}{x+4} & \quad \text{B}
\end{align*}
\]

\[
\frac{5}{x+5} \times \frac{4}{x+4} = \frac{5}{33}
\]

\[
\frac{20}{(x+5)(x+4)} = \frac{5}{33}
\]

\[
660 = 5(x+5)(x+4)
\]

\[
132 = (x+5)(x+4)
\]

\[
132 = x^2 + 9x + 20
\]

\[
0 = x^2 + 9x - 112
\]

\[
0 = (x+16)(x-7)
\]

\[
x = -16 \quad x = 7
\]

\[
\therefore \quad x = 7
\]

(Total for question 5 is 7 marks)
There are \( n \) counters in a bag. 4 of the counters are red and the rest are blue.

Ross takes a counter from the bag at random and does not replace it. He then takes another counter at random from the bag.

The probability that Ross takes two blue counters is \( \frac{1}{3} \)

(a) Show that \( n^2 - 13n + 30 = 0 \)

\[
\frac{n-4}{n} \times \frac{n-5}{n-1} = \frac{1}{3}
\]

\[
\frac{(n-4)(n-5)}{n(n-1)} = \frac{1}{3}
\]

\[
\frac{n^2 - 5n - 4n + 20}{n^2 - n} = \frac{1}{3}
\]

\[
\frac{n^2 - 9n + 20}{n^2 - n} = \frac{1}{3}
\]

\[
3(n^2 - 9n + 20) = n^2 - n
\]

\[
3n^2 - 27n + 60 = n^2 - n
\]

\[
2n^2 - 26n + 60 = 0
\]

\[
n^2 - 13n + 30 = 0
\]

\[
(n - 3)(n - 10) = 0
\]

\[
n = 3 \quad n = 10
\]

\[n \text{ cannot be less than } 4 \text{ so } n = 10\]

(b) Find the value of \( n \).

\[
n^2 - 13n + 30 = 0
\]

\[
(n - 3)(n - 10) = 0
\]

\[
n = 3 \quad n = 10
\]

\[n \text{ cannot be less than } 4 \text{ so } n = 10\]

(Total for question 6 is 7 marks)
7 There are $n$ counters in a bag.
8 of the counters are red and the rest are blue.

Adam takes a counter from the bag at random and does not replace it. He then takes another counter at random from the bag.

The probability that Adam takes two blue counters is $\frac{1}{5}$

(a) Show that $n^2 - 21n + 90 = 0$

$\frac{n-8}{n} \times \frac{n-9}{n-1} = \frac{1}{5}$

$\frac{(n-8)(n-9)}{n(n-1)} = \frac{1}{5}$

$\frac{n^2 - 9n - 8n + 72}{n^2 - n} = \frac{1}{5}$

$\frac{n^2 - 17n + 72}{n^2 - n} = \frac{1}{5}$

$5(n^2 - 17n + 72) = n^2 - n$

$5n^2 - 85n + 360 = n^2 - n$

$4n^2 - 84n + 360 = 0$

$n^2 - 21n + 90 = 0$

(b) Find the value of $n$.

$(n - 6)(n - 15) = 0$

$n = 6 \quad n = 15$

$n$ cannot be less than 8 : $n = 15$

15

(Total for question 1 is 7 marks)