Write your name here

Surname	Other Names

Mathematics

June 2024 Practice Paper 3 (Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- **Calculators may be used**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2}(a + b)\, h$$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

$$\text{Circumference of a circle} = 2\pi r = \pi d$$

$$\text{Area of a circle} = \pi r^2$$

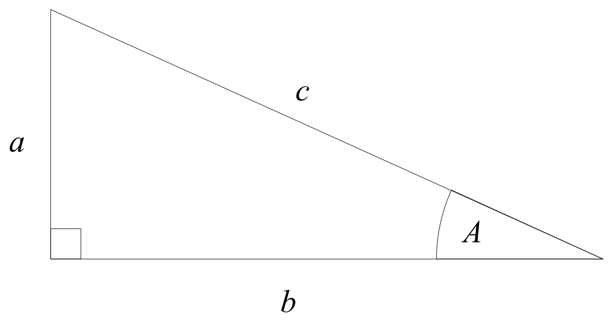
Quadratic formula

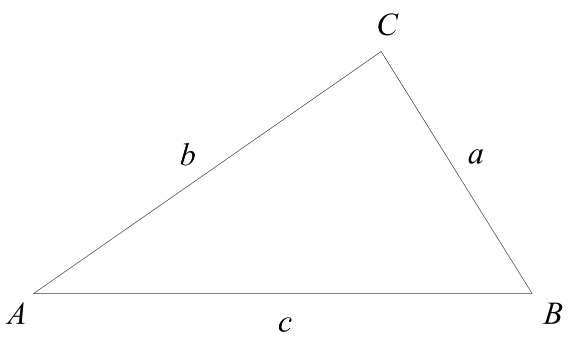
The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry





In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

In any triangle ABC where a, b and c are the length of the sides:

sine rule: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

P (A or B) = P (A) + P (B) – P (A and B)

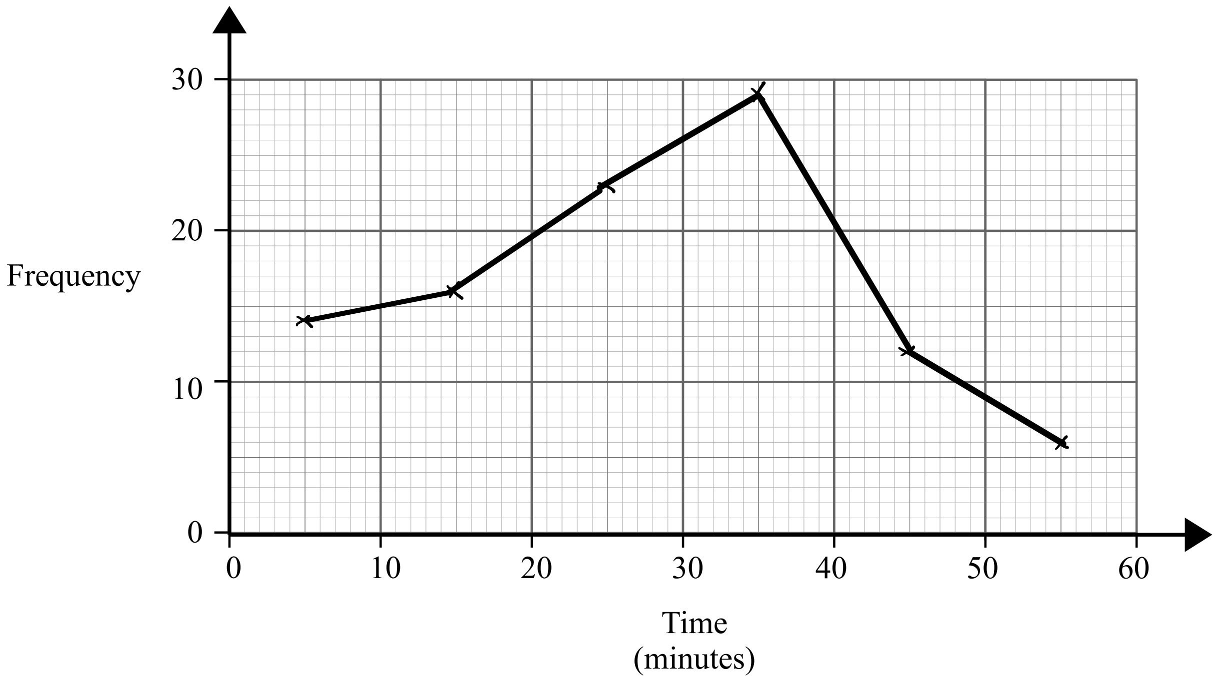
P (A and B) = P (A given B) P(B)

END OF EXAM AID

1 The frequency table shows the time taken for 100 people to travel to an event.

Time (minutes)	**Frequency**
$0 < t \leqslant 10$	14
$10 < t \leqslant 20$	16
$20 < t \leqslant 30$	23
$30 < t \leqslant 40$	29
$40 < t \leqslant 50$	12
$50 < t \leqslant 60$	6

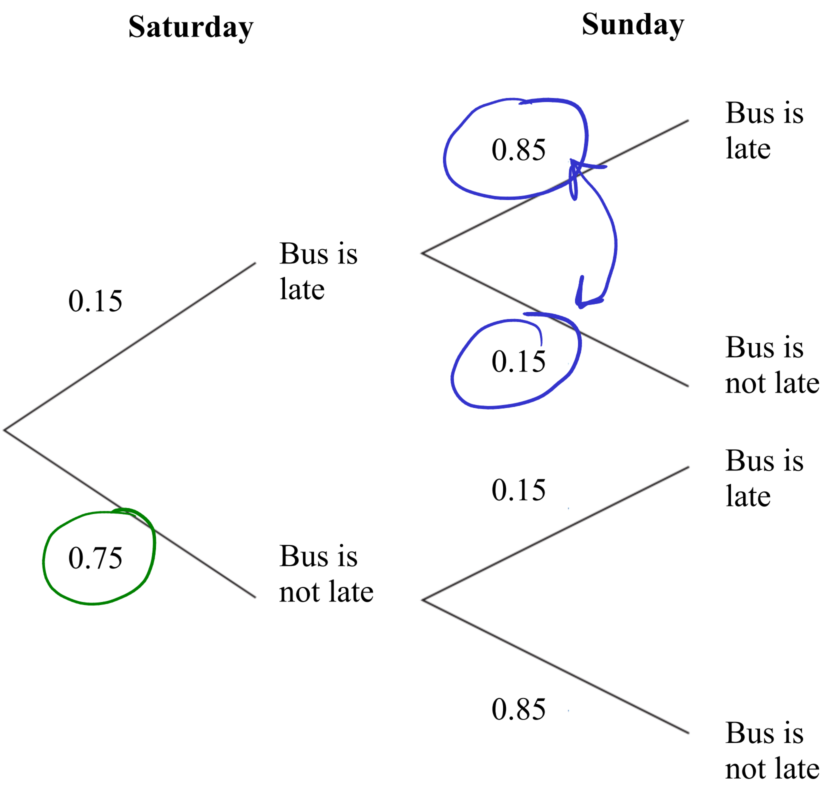
Draw a frequency polygon to show this information.



(Total for Question 1 is 2 marks)

2 Bradley gets the bus on Saturday and Sunday.
The probability that Bradley's bus will be late on any day is 0.15

Bradley draws this probability tree diagram.
The diagram is not correct.



Write down two things that are wrong with the probability tree diagram.

1 The probability the bus is not Late on Saturday should be 0.85

2 The probability of the bus being late on Sunday (after being late on Saturday) should be 0.15

(Total for Question 2 is 2 marks)

3 Matt wants to invest £8000 for three years. He can choose between Bank A and Bank B.

Bank A	**Bank B**
1.2% compound interest per annum	2% compound interest in the first year 1% compound interest for each extra year

Which bank will give Matt the most interest after three years.
You must show your working.

Bank A

8000×1.012^3

$= 8291.47$

Bank B

$8000 \times 1.02 \times 1.01^2$

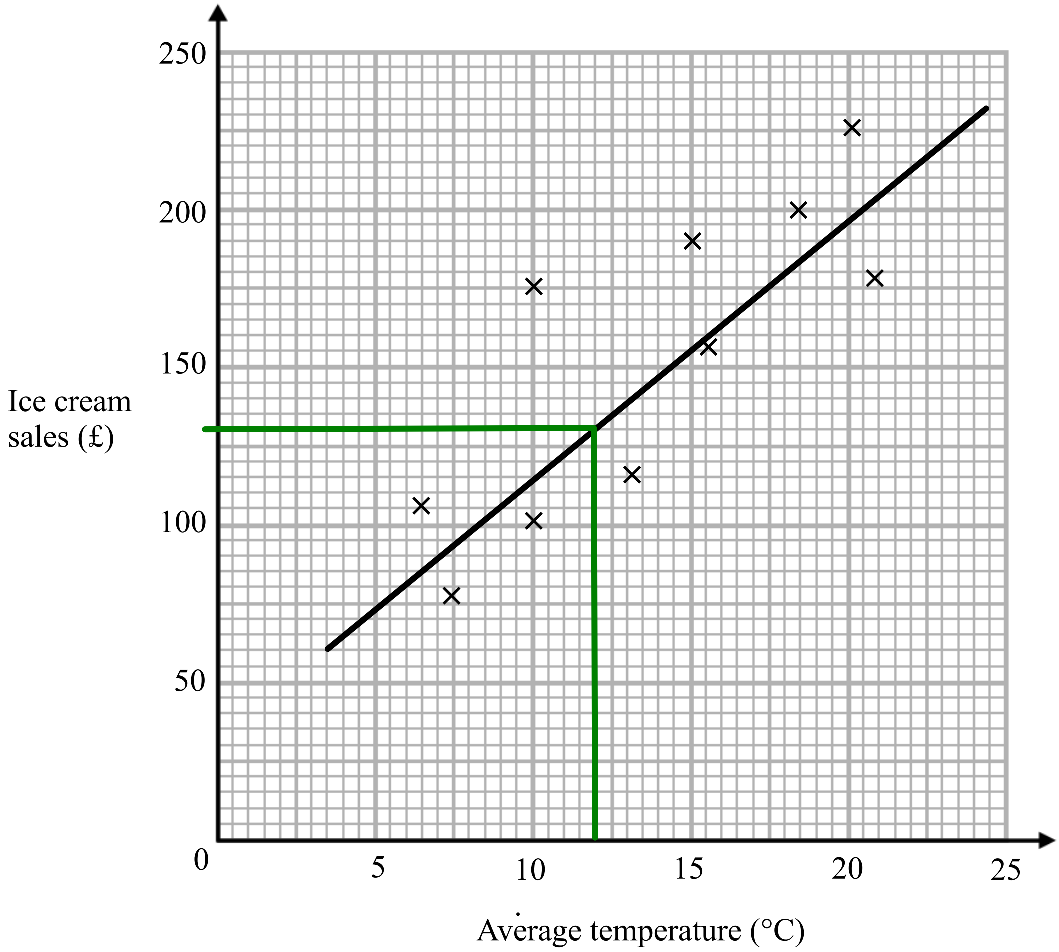
$= 8324.02$

Bank B

(Total for Question 3 is 4 marks)

4 The average daytime temperature for 10 days is recorded.
A shop also records its ice cream sales for each of the 10 days.

The scatter graph shows this information.



(a) What type of correlation does the scatter graph show?

positive

(1)

(b) On the 11th day the temperature was 12°C.
Estimate the ice cream sales on the 11th day.

£ 130

(2)

(c) The shop's manager wants to use the scatter graph to predict the ice cream sales for a day with an average temperature of 2°C. Comment on the reliability of this prediction.

It would not be reliable. 2°C is outside the range of data.

(1)

(Total for Question 4 is 4 marks)

5 Find 5% of 3.8×10^{105}
Give your answer in standard form

5% of 3.8

$0.05 \times 3.8 = 0.19$

0.19×10^{105}
1.9×10^{104}

1.9×10^{104}

(Total for Question 5 is 3 marks)

6 Verity buys 12 bottles of apple juice for a total cost of £15
Verity sells all 12 bottles at £1.75 each bottle.

$\frac{\text{change}}{\text{original}} \times 100$

Work out Verity's percentage profit.

$12 \times 1.75 = 21$

$\frac{21 - 15}{15} \times 100 = \underline{\underline{40}}$

40 %

(Total for Question 6 is 3 marks)

7 $y^2 \times y^a = y^7$

$2 + a = 7$

(a) Find the value of a.

5

(1)

$(y^4)^b = y^{12}$

(b) Find the value of b.

$4 \times b = 12$

3

(1)

(Total for Question 7 is 2 marks)

---

8 Change a speed of 81 kilometres per hour to a speed in metres per second

81 km per hour

81 000 m per hour

$\div 60$

1350 m per minute

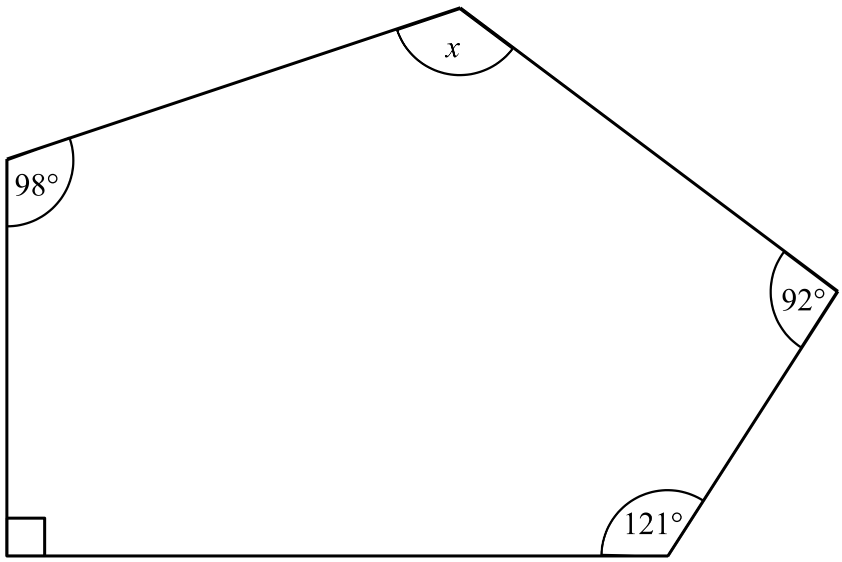
$\div 60$

22.5 m per second

22.5 m/s

(Total for Question 8 is 3 marks)

9 The diagram shows a pentagon.



Work out the value of x

Angles in pentagon = 540° (3×180)

540 − 90 − 98 − 92 − 121

= 139°

139°

(Total for Question 9 is 3 marks)

10 The density of orange cordial is 1.21 grams per cm^3.

The density of carbonated water is 1.01 grams per cm^3.

A drink with a volume of 280 cm^3 is made by mixing 1 part of orange cordial with 7 parts of carbonated water.

Work out the density of the drink.

$\text{density} = \frac{\text{mass}}{\text{volume}}$

1 : 7 (8 parts)

$\frac{280}{8} = 35\ cm^3$ per part

35 : 245

orange water

mass = density × volume

orange: mass = 1.21 × 35
= 42.35g

water: mass = 1.01 × 245
= 247.45g

$\text{density} = \frac{\text{total mass}}{\text{total volume}} = \frac{42.35 + 247.45}{280}$

$= 1.035$

1.035 g/cm^3

(Total for Question 10 is 4 marks)

11 There are 5 starters, 6 main courses and x desserts in a restaurant.

Riley says there are 130 different ways of choosing a starter, a main course and a dessert.

Could Riley be correct?
You must show your working.

$5 \times 6 \times x = 30x$

$30x = 130$
$x = 4.\dot{3}$

No. Riley cannot be correct.
x must be a whole number.

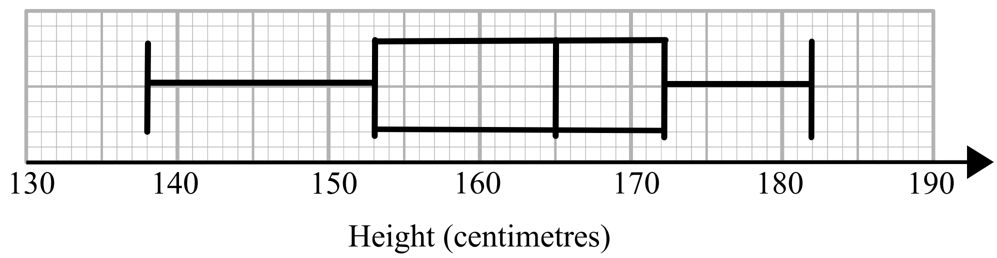
(Total for Question 11 is 2 marks)

---

12 Holly recorded the heights, in centimetres, of some girls.
She used her results to work out the information in this table.

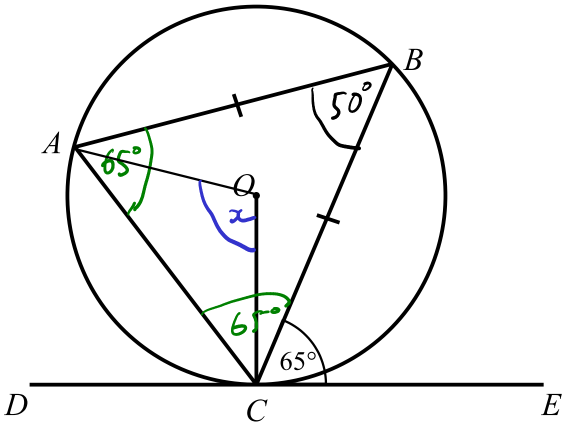
Least height	138 cm
Interquartile range	19 cm
Median	165 cm
Upper quartile	172 cm
Range	44 cm

Draw a box plot for the information in the table.



(Total for Question 12 is 2 marks)

13



A, B and *C* are points on the circumference of a circle, centre *O*.
DCE is a tangent to the circle.

$AB = BC$
Angle $BCE = 65°$

Find the size of angle *AOC*.
You must show all your working.

ABC = 65° Alternate segment theorem

ACB = 65° Angles at the base of an isosceles triangle are equal

ABC = 50° Angles in a triangle add to 180°

AOC = 100° Angle at centre is twice angle at circumference.

100 °

(Total for Question 13 is 4 marks)

14 Make d the subject of $e = \sqrt{\dfrac{d + e}{de - 2f}}$

$$e^2 = \frac{d+e}{de-2f}$$

$$e^2(de - 2f) = d + e$$

$$de^3 - 2e^2f = d + e$$

$$de^3 = d + e + 2e^2f$$

$$de^3 - d = e + 2e^2f$$

$$d(e^3 - 1) = e + 2e^2f$$

$$d = \frac{e + 2e^2f}{e^3 - 1}$$

(Total for Question 14 is 4 marks)

---

15 Here are the first five terms of a quadratic sequence. $an^2 + bn + c$

-3 4 14 27 43

Find an expression, in terms of n, for the nth term of this sequence

$a+b+c \rightarrow -3$ 4 14 27 43

$3a+b \rightarrow 7$ 10 13 16

$2a \rightarrow 3$ 3 3

$2a = 3$
$a = 1.5$

$3(1.5) + b = 7$
$4.5 + b = 7$
$b = 2.5$

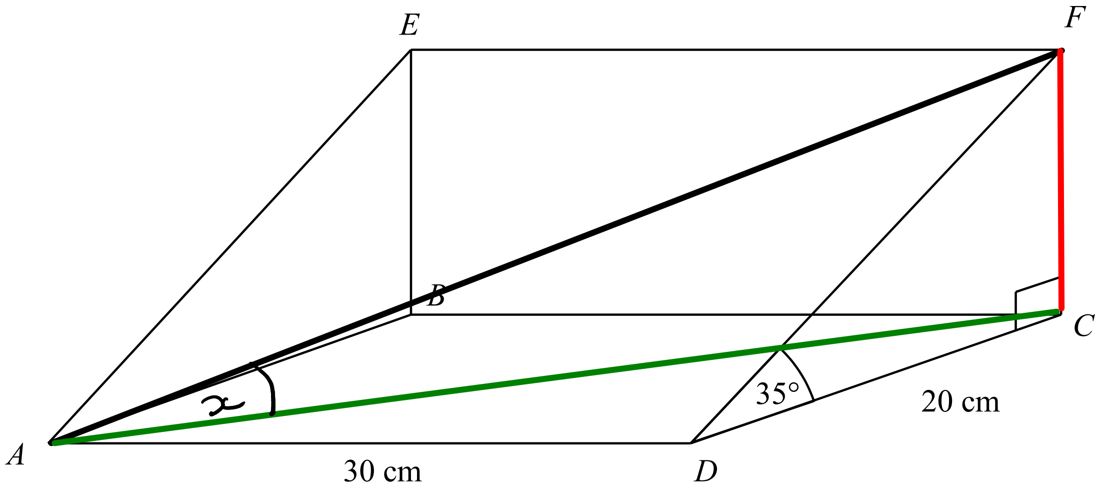
$1.5 + 2.5 + c = -3$
$4 + c = -3$
$c = -7$

$1.5n^2 + 2.5n - 7$

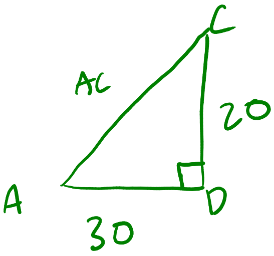
(Total for Question 15 is 3 marks)

16 The diagram shows a triangular prism.

$CD = 20$ cm
$AD = 30$ cm
Angle $FDC = 35°$



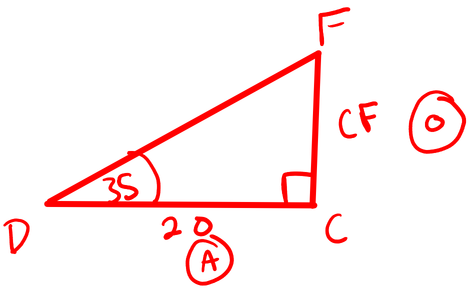
Calculate the size of the angle the line AF makes with the plane $ABCD$.
Give your answer correct to 3 significant figures.



$AC^2 = 20^2 + 30^2$

$AC = \sqrt{20^2 + 30^2}$

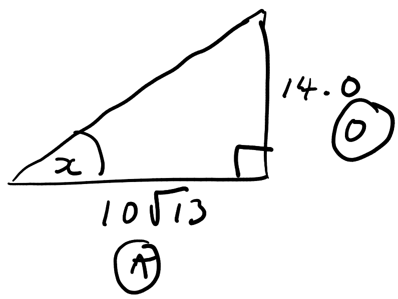
$= 10\sqrt{13}$



$\tan(35) = \frac{CF}{20}$

$CF = 20\tan(35)$

$= 14.0$



$\tan x = \frac{14.0}{10\sqrt{13}}$

$= 21.2°$

21.2 °

(Total for Question 16 is 4 marks)

17 Prove algebraically that the sum of the squares of any 2 odd positive integers is always even.

$2n+1$ and $2m+1$

$(2n+1)^2 + (2m+1)^2$

$(2n+1)(2n+1) + (2m+1)(2m+1)$

$4n^2 + 2n + 2n + 1 + 4m^2 + 2m + 2m + 1$

$4n^2 + 4m^2 + 4n + 4m + 2$

$2(2n^2 + 2m^2 + 2n + 2m + 1)$

multiple of 2 $\therefore$ even

(Total for Question 17 is 3 marks)

18 The functions f and g are such that

$$f(x) = \frac{3}{6x + 5} \qquad x \neq -\frac{5}{6}$$

$$g(x) = x^2 - 2 \qquad x \geq 0$$

Solve fg(x) = 1

$$\frac{3}{6(x^2-2)+5} = 1$$

$$\frac{3}{6x^2-12+5} = 1$$

$$\frac{3}{6x^2-7} = 1$$

$$3 = 6x^2 - 7$$

$$10 = 6x^2$$

$$\frac{10}{6} = x^2$$

$$x = \sqrt{\frac{10}{6}}$$

$$= \frac{\sqrt{15}}{3}$$

$\frac{\sqrt{15}}{3}$ ....

(Total for Question 18 is 3 marks)

19 (a) Show that the equation $x^4 - 3x^3 - 7 = 0$ can be written in the form $x = \sqrt[4]{3x^3 + 7}$

$x^4 = 3x^3 + 7$

$x = \sqrt[4]{3x^3 + 7}$

(1)

(b) Starting with $x_0 = 3$
use the iteration formula $x_{n+1} = \sqrt[4]{3x_n^3 + 7}$ three times to find an estimate for a solution to $x^4 - 3x^3 - 7 = 0$

$x_1 = \sqrt[4]{3(3)^3 + 7} = 3.0628$

$x_2 = \sqrt[4]{3(3.0628)^3 + 7} = 3.10705$

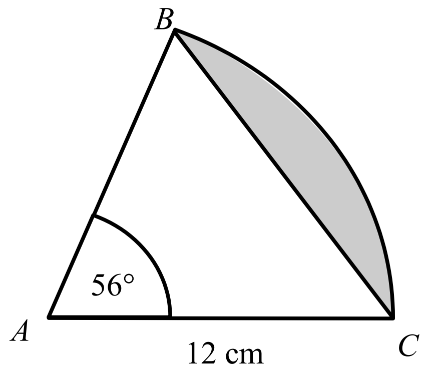
$x_3 = \sqrt[4]{3(\text{Ans})^3 + 7} = 3.13816$

3.13816

(3)

(Total for Question 19 is 4 marks)

20 BAC is a sector of a circle, centre A.
AC is 12 cm.



Find the area of the shaded segment.
Give your answer to 3 significant figures.

Area of sector $= \frac{56}{360} \times \pi(12)^2$

$= 70.371675\ldots \text{ cm}^2$

Area of triangle $= \frac{1}{2} ab \sin C$

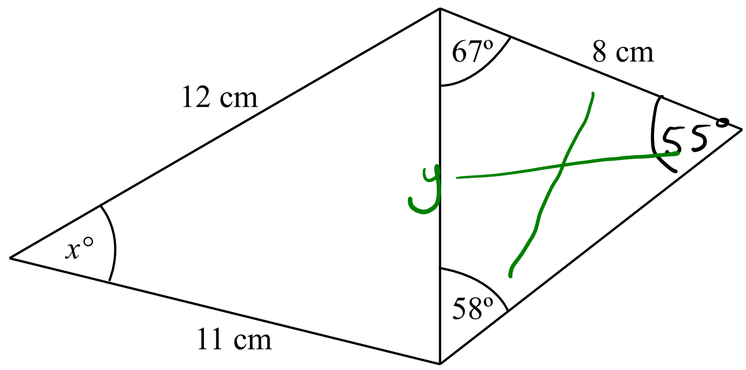
$= \frac{1}{2}(12)(12)\sin(56)$

$= 59.6907\ldots \text{ cm}^2$

$70.37 - 59.69 = 10.7$ ....10.7.......... cm^2

(Total for Question 20 is 3 marks)

21



$180 - 67 - 58$

Work out the value of *x*.
Give your answer to 1 decimal place.

$$\frac{y}{\sin 55} = \frac{8}{\sin 58}$$

$$y = \frac{8}{\sin 58} \times \sin 55$$

$$= 7.727 \text{ cm}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{11^2 + 12^2 - 7.727^2}{2(11)(12)}$$

$$= \underline{39.0°}$$

39.0

(Total for Question 21 is 4 marks)

22 Solve $\frac{1}{1-2x} + \frac{2}{x+3} = 3$

$$\frac{(x+3)}{(1-2x)(x+3)} + \frac{2(1-2x)}{(1-2x)(x+3)} = 3$$

$$\frac{x+3+2(1-2x)}{(1-2x)(x+3)} = 3$$

$$x+3+2(1-2x) = 3(1-2x)(x+3)$$

$$x+3+2-4x = 3(x+3-2x^2-6x)$$

$$5-3x = 3x+9-6x^2-18x$$

$$6x^2+12x-4 = 0$$

$$3x^2+6x-2 = 0$$

$a=3 \quad b=6 \quad c=-2$

$$x = \frac{-(6) \pm \sqrt{(6)^2-4(3)(-2)}}{2(3)}$$

$$= 0.291 \quad \text{or} \quad -2.29$$

$x = 0.291$ or -2.29

(Total for Question 22 is 5 marks)

23 Prove algebraically that the straight line with equation $3x - 2y + 13 = 0$ is a tangent to the circle with equation $x^2 + y^2 = 13$

Sim. eq.

$3x + 13 = 2y$

$$\boxed{y = \frac{3x+13}{2}}$$

Sub. into $x^2+y^2=13$

$$x^2 + \left(\frac{3x+13}{2}\right)^2 = 13$$

$$x^2 + \frac{(3x+13)}{2} \times \frac{(3x+13)}{2} = 13$$

$$x^2 + \frac{9x^2+39x+39x+169}{4} = 13$$

$$4x^2 + 9x^2 + 78x + 169 = 52$$

$$13x^2 + 78x + 117 = 0$$

$$x^2 + 6x + 9 = 0$$

$$(x+3)(x+3) = 0$$

$\underline{\underline{x = -3}}$ only one point of intersection

$\therefore$ must be a tangent.

(Total for Question 23 is 5 marks)

24 The displacement of an object, s, is given by the formula

$$s = \frac{v^2 - u^2}{2a}$$

where,

$v = 15.49$ correct to 2 decimal places
$u = 4.92$ correct to 3 significant figures
$a = 7.5$ correct to 2 significant figures

By considering bounds, work out the value of s to a suitable degree of accuracy.
Show your working clearly and give a reason for your answer.

$15.485 \leqslant v < 15.495$
$4.915 \leqslant u < 4.925$
$7.45 \leqslant a < 7.55$

upper bound $s = \dfrac{15.495^2 - 4.915^2}{2(7.45)}$

$= 14.4924698$

Lower bound $s = \dfrac{15.485^2 - 4.925^2}{2(7.55)}$

$= 14.27348344$

14 2 significant figures

both upper and lower bounds round to 14 to 2 s.f.

.... 14 ....

(Total for Question 24 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS