

Write your name here

Surname

Other Names

Mathematics

June 2024 Practice Paper 2 (Calculator) Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- **Calculators may be used.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working.**



Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2}(a + b)h$$

Volume of a prism = area of cross section \times length

Where r is the radius and d is the diameter:

$$\text{Circumference of a circle} = 2\pi r = \pi d$$

$$\text{Area of a circle} = \pi r^2$$

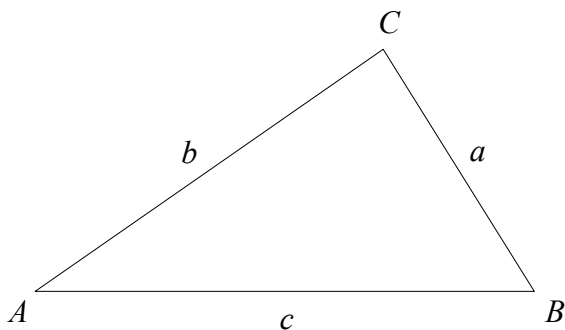
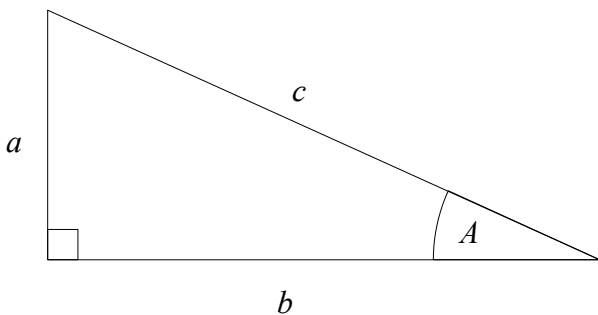
Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry



In any right-angled triangle where a , b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a , b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \quad \cos A = \frac{b}{c} \quad \tan A = \frac{a}{b}$$

In any triangle ABC where a , b and c are the length of the sides:

$$\text{sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of triangle} = \frac{1}{2}ab \sin C$$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P \left(1 + \frac{r}{100} \right)^n$$

Probability

Where $P(A)$ is the probability of outcome A and $P(B)$ is the probability of outcome B :

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A \text{ given } B) P(B)$$

END OF EXAM AID

1 The height of a building is 310 metres, correct to the nearest metre.

Complete the error interval for the height of the building.

$$\underline{309.5} \text{ m} \leq \text{length} < \underline{310.5} \text{ m}$$

(Total for Question 1 is 2 marks)

2 Work out $(3.12 \times 10^{-6}) \div (2.5 \times 10^{-4})$

Give your answer in standard form.

$$0.01248$$

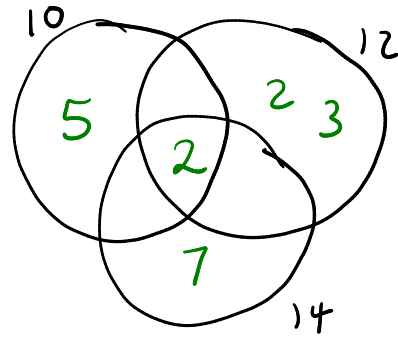
$$1.248 \times 10^{-2}$$

$$\underline{1.248 \times 10^{-2}}$$

(Total for Question 2 is 2 marks)

3 Three buses, bus A, bus B and bus C, all use the same bus stop.

Bus A runs every 10 minutes. 2×5
 Bus B runs every 12 minutes. $2 \times 2 \times 3$
 Bus C runs every 14 minutes. 2×7



All three buses are at the bus stop at 11 am.

What time will all three buses next be at the bus stop.

$$\text{LCM} = 2 \times 2 \times 3 \times 5 \times 7$$

$$= 420$$

$$420 \text{ minutes} \quad \frac{420}{60} = 7 \text{ hours}$$

6 pm

(Total for Question 3 is 3 marks)

4 The table gives information about the times taken, in seconds, by 20 students to run a race.

Time (t seconds)	Midpoint	Frequency	Midpoint x Freq
$20 < t \leq 25$	22.5	x 2	45
$25 < t \leq 30$	27.5	x 10	275
$30 < t \leq 35$	32.5	x 5	162.5
$35 < t \leq 45$	40	x 3	120
		<u>20</u>	<u>602.5</u>

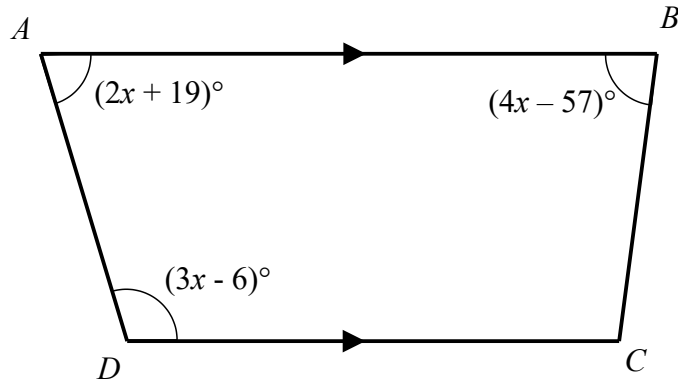
Work out an estimate for the mean time.

$$\frac{602.5}{20} = 30.125$$

30.1 seconds

(Total for Question 4 is 3 marks)

5 $ABCD$ is a trapezium.



AB is parallel to DC
Find the size of angle BCD .

Co-interior angles add to 180°

$$2x + 19 + 3x - 6 = 180$$

$$5x + 13 = 180$$

$$5x = 167$$

$$x = 33.4$$

$$\begin{aligned} \angle B &= 4(33.4) - 57 \\ &= 76.6^\circ \end{aligned}$$

$$\begin{aligned} \angle BCD &= 180 - 76.6 \\ &= \underline{103.4^\circ} \end{aligned}$$

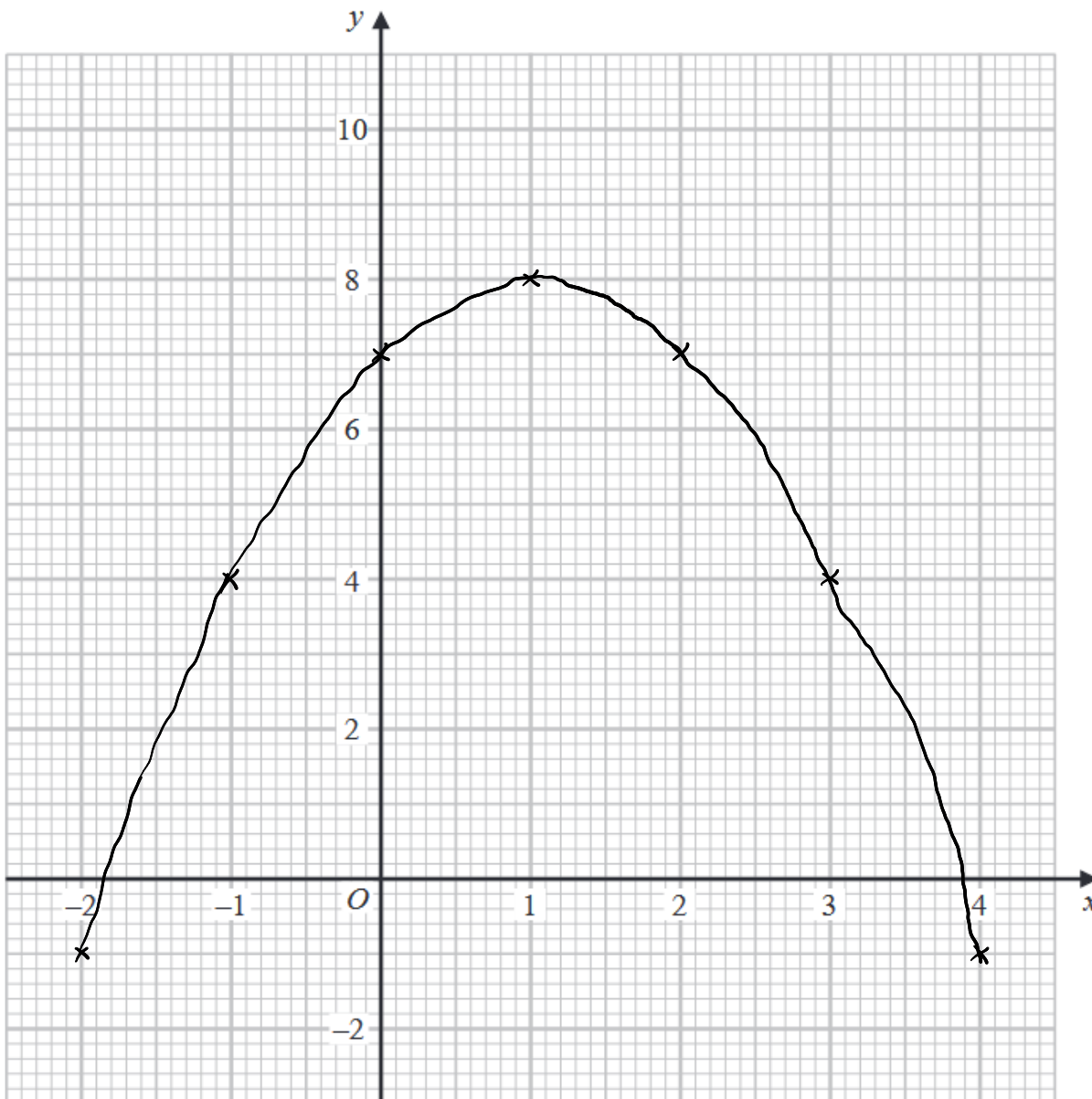
.....103.4°

(Total for Question 5 is 4 marks)

6 (a) Complete the table of values for $y = 7 + 2x - x^2$

x	-2	-1	0	1	2	3	4
y	-1	4	7	8	7	4	-1

(b) On the grid, draw the graph of $y = 7 + 2x - x^2$ for values of x from -2 to 4 (2)



(c) Use your graph to find estimates of the solutions of the equation $7 + 2x - x^2 = 0$

either -1.8 or -1.9 and 3.8 or 3.9

(2)

(Total for Question 6 is 6 marks)

7

Josh drove 39 miles from Luton to Cambridge.
He then drove 63 miles from Cambridge to Norwich.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

Josh's average speed from Luton to Cambridge was 32 miles per hour
Josh took 80 minutes to drive from Cambridge to Norwich.

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Work out Josh's average speed for his total drive from Luton to Norwich.

L → C

C → N

$$\text{Time} = \frac{39}{32} = 1.21875 \text{ hours}$$

$$\text{Time} = 80 \text{ mins}$$

$$\frac{80}{60} = \frac{4}{3} \text{ or } 1.\dot{3} \text{ hours}$$

$$\begin{aligned} \text{Total time} &= \frac{39}{32} + \frac{4}{3} \\ &= 2.552083\dots \end{aligned}$$

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance}}{\text{Total time}} \\ &= \frac{39 + 63}{2.55208\dots} \\ &= 39.96734\dots \end{aligned}$$

..... 40.0 miles per hour

(Total for Question 7 is 4 marks)

- 8 Milly invests £2000 in a savings account for 4 years.
She gets 3.9% per year compound interest.

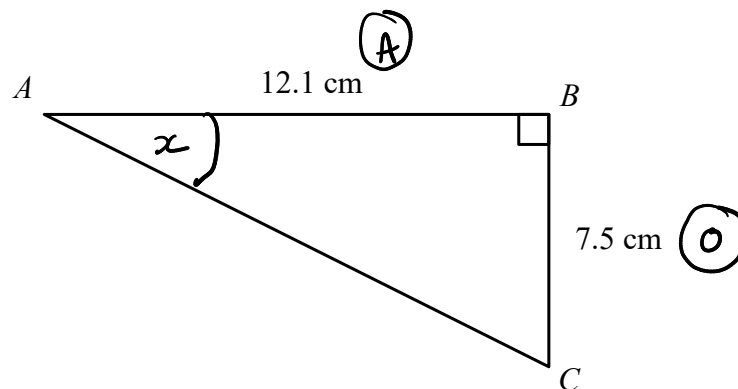
Work out how much money Milly will have in her savings account at the end of 4 years.
Give your answer correct to the nearest pound.

$$2000 \times 1.039^4$$
$$= 2330.73$$

£ 2331

(Total for Question 8 is 3 marks)

9



Work out the size of angle BAC .
Give your answer correct to 3 significant figures.

$$\tan x = \frac{7.5}{12.1}$$

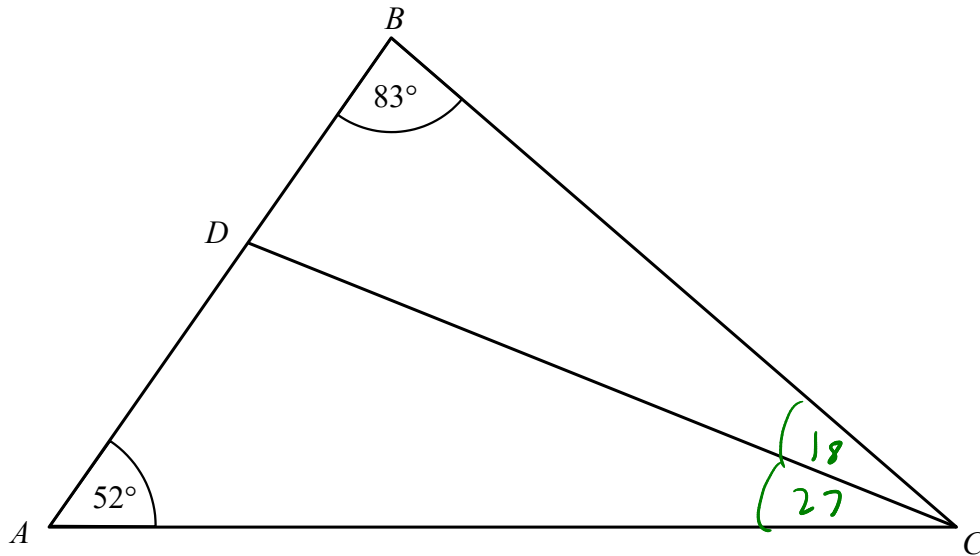
$$x = \tan^{-1} \left(\frac{7.5}{12.1} \right)$$

$$= 31.8$$

31.8 °

(Total for Question 9 is 2 marks)

10 The diagram shows triangle ABC .



ADB is a straight line.

the size of angle BCD : the size of angle ACD = 2 : 3

Work out the size of angle ADC .

$$180 - 83 - 52 = 45$$

$$\boxed{9 \quad 9} : \boxed{9 \quad 9 \quad 9}$$

$$\frac{45}{5} = 9 \quad 18 : 27$$

$$180 - 27 - 52 = \underline{101^\circ}$$

..... 101 °

(Total for Question 10 is 4 marks)

- 11 A company has 9 employees available to complete a job.
It is known that 7 employees can complete the job in 12 days.

Davina says that the 9 employees will be able to complete the job in less than 10 days.

Is Davina correct?

You must show all your working.

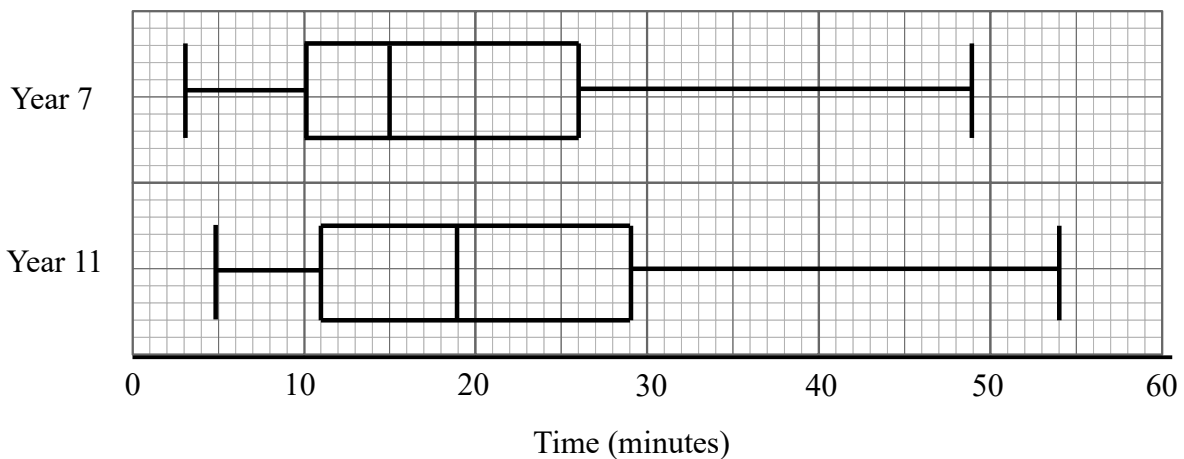
$$7 \times 12 = 84$$

$$84 \div 9 = 9.\dot{3} \text{ days}$$

$$\underline{9.\dot{3}} < 10 \quad \underline{\text{Yes}}$$

(Total for Question 11 is 3 marks)

- 12 The box plots show the time it took year 7 and year 11 students to travel to school on one day.



Compare the distribution of the times it took the year 7 and year 11 students to travel to school.

- The median time taken for year 7 students is lower
- The inter quartile range of year 11 students times is greater

(Total for Question 12 is 2 marks)

13 In a restaurant there are 7 starters, 9 main courses and 6 desserts on the menu.

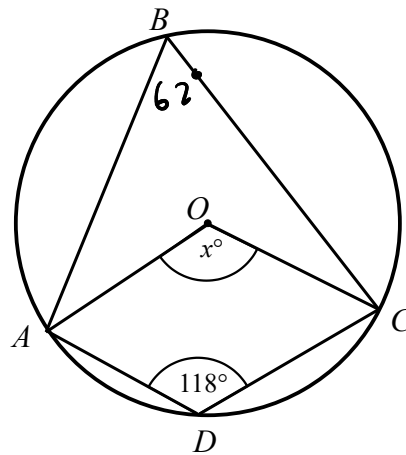
Work out the total number of ways of choosing a starter, a main course and a dessert.

$$7 \times 9 \times 6$$

378

(Total for Question 13 is 2 marks)

14



A , B , C and D are points on the circumference of a circle, centre O .

Angle $ADC = 118^\circ$

Angle $AOC = x^\circ$

Work out the value of x .

You must show all your working.

$$180 - 118 = 62$$

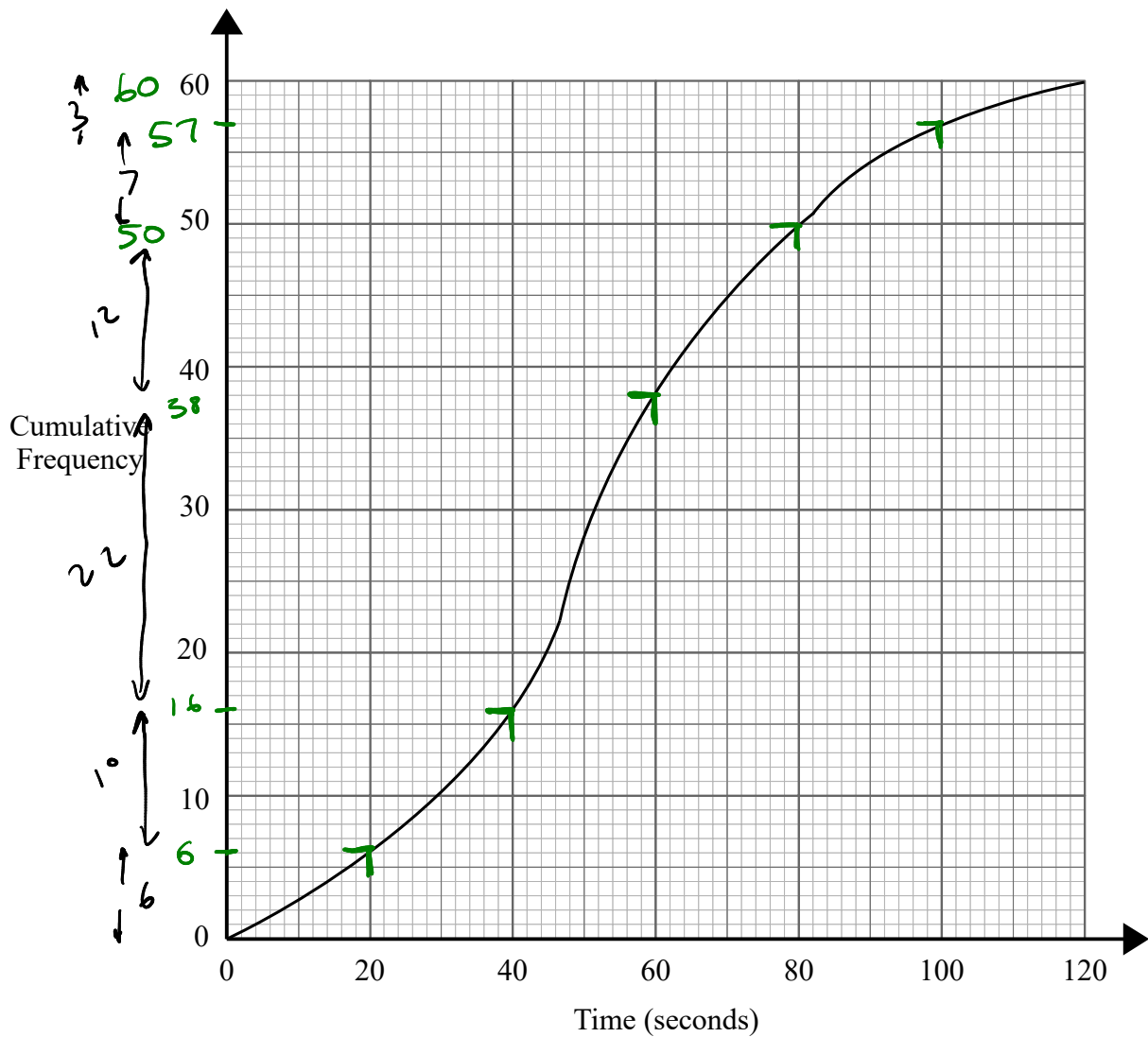
(opposite angles in cyclic quad. add to 180°)

$$\begin{aligned} x &= 62 \times 2 \\ &= 124^\circ \end{aligned}$$

Angle at centre is twice angle at circumference
124°

(Total for Question 14 is 3 marks)

- 15 The cumulative frequency graph gives information about the time, in seconds, each of 60 people took to complete a puzzle.



Use the graph to complete the frequency table to give information about the time, t seconds, each of the 60 people took to complete the puzzle.

Time taken	Frequency
$0 < t \leq 20$	6
$20 < t \leq 40$	10
$40 < t \leq 60$	22
$60 < t \leq 80$	12
$80 < t \leq 100$	7
$100 < t \leq 120$	3

(Total for Question 15 is 2 marks)

- 16 Here are 9 cards.
Each card has a number on it.



Emma takes two cards at random.

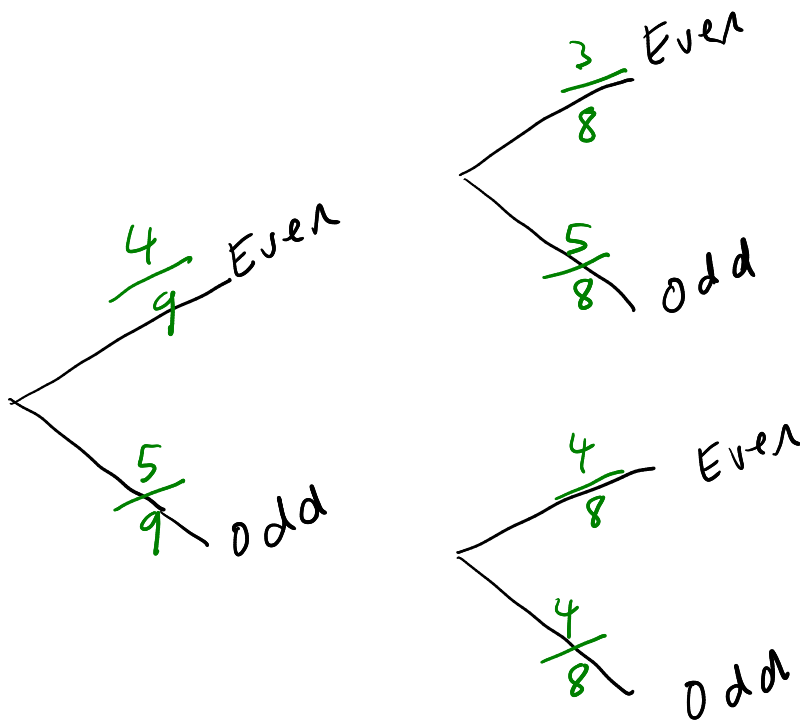
Work out the probability that the sum of the two cards is an odd number.

$$\text{Even} + \text{even} = \text{even}$$

$$\text{odd} + \text{odd} = \text{even}$$

$$\text{odd} + \text{even} = \text{odd}$$

$$\text{even} + \text{odd} = \text{odd}$$



$$P(\text{Even, Odd}) = \frac{4}{9} \times \frac{5}{8} = \frac{20}{72}$$

$$P(\text{Odd, Even}) = \frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$$

$$\frac{20}{72} + \frac{20}{72} = \frac{40}{72}$$

$$\frac{40}{72}$$

(Total for Question 16 is 3 marks)

17 (a) Using $x_{n+1} = \sqrt{6x_n - 1}$

with $x_0 = 5$

(a) Find the values of x_1 , x_2 and x_3

$$x_1 = \sqrt{6(5) - 1} = \sqrt{29} = 5.3851648$$

$$x_2 = \sqrt{6(\text{Ans}) - 1}$$

$$x_3 = \sqrt{6(\text{Ans}) - 1}$$

$$x_1 = \underline{5.385164807}$$

$$x_2 = \underline{5.595622293}$$

$$x_3 = \underline{5.70734034}$$

(3)

(b) Explain the relationship between the values of x_1 , x_2 and x_3 and the equation $x^2 - 6x + 1 = 0$

$$x^2 - 6x + 1 = 0$$

$$x^2 = 6x - 1$$

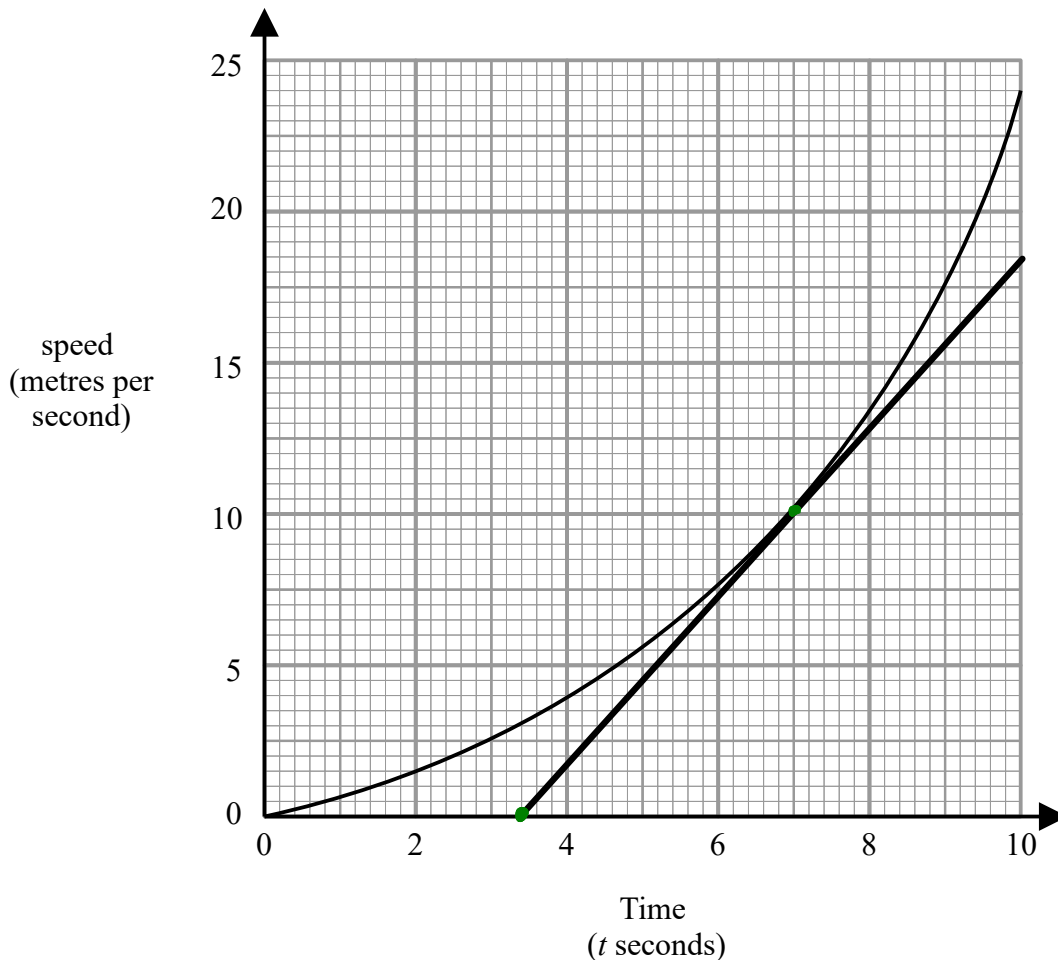
$$x = \sqrt{6x - 1}$$

They are estimates or the solutions to the equation.

(2)

(Total for Question 17 is 5 marks)

- 18 The graph below shows the speed of a car, in metres per second, t seconds after it starts moving.



- (a) Calculate an estimate for the gradient of the graph when $t = 7$
You must show how you get your answer.

$$(3.4, 0) \quad (7, 10)$$

$$\frac{10}{3.6} = 2.7$$

$$\underline{\underline{2.7}}$$

(3)

- (b) Describe what the gradient in part (a) represents.

The acceleration.

(1)

(Total for Question 18 is 4 marks)

- 19 Show that $\frac{2x}{x-3} - \frac{3x-1}{x+3} + 1$ can be written in the form $\frac{ax+b}{x^2-9}$ where a and b are integers.

$$\frac{2x(x+3)}{(x+3)(x-3)} - \frac{(3x-1)(x-3)}{(x+3)(x-3)} + \frac{(x+3)(x-3)}{(x+3)(x-3)}$$

$$\frac{2x^2 + 6x}{x^2 - 9} - \frac{3x^2 - 9x - x + 3}{x^2 - 9} + \frac{x^2 - 9}{x^2 - 9}$$

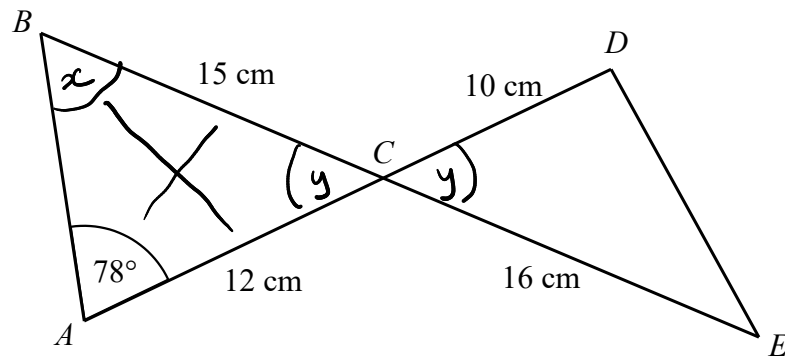
$$\frac{2x^2 + 6x - (3x^2 - 10x + 3) + x^2 - 9}{x^2 - 9}$$

$$\frac{\cancel{2x^2} + 6x - \cancel{3x^2} + 10x - 3 + \cancel{x^2} - 9}{x^2 - 9}$$

$$\frac{16x - 12}{x^2 - 9}$$

(Total for Question 19 is 4 marks)

- 20 Here is a shape formed from two triangles ABC and CDE
 ACD and BCE are straight lines.



$AC = 12 \text{ cm}$ $BC = 15 \text{ cm}$ $CE = 16 \text{ cm}$ $CD = 10 \text{ cm}$
 Angle $BAC = 78^\circ$

Work out the length of DE
 Give your answer correct to 3 significant figures.

$$\frac{\sin x}{12} = \frac{\sin 78}{15}$$

$$\sin x = \frac{\sin 78}{15} \times 12$$

$$= 0.7825\dots$$

$$x = \sin^{-1}(0.7825\dots)$$

$$= 51.4917\dots$$

$$y = 180 - 78 - 51.4917$$

$$= 50.5082898$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 10^2 + 16^2 - 2(10)(16) \cos(50.508)$$

$$= 152.49$$

$$a = 12.3487$$

12.3

cm

(Total for Question 20 is 5 marks)

21 Solve algebraically the simultaneous equations

$$\begin{aligned}x^2 + 2y^2 &= 10 \\ 3x - 2y &= 8\end{aligned}$$

$$3x = 2y + 8$$

$$x = \frac{2y + 8}{3}$$

$$\left(\frac{2y + 8}{3}\right)^2 + 2y^2 = 10$$

$$\frac{(2y + 8)(2y + 8)}{9} + 2y^2 = 10$$

$$\frac{4y^2 + 16y + 16y + 64}{9} + 2y^2 = 10$$

$$4y^2 + 32y + 64 + 18y^2 = 90$$

$$22y^2 + 32y - 26 = 0$$

$$11y^2 + 16y - 13 = 0$$

$$a = 11 \quad b = 16 \quad c = -13$$

$$y = \frac{-16 \pm \sqrt{(16)^2 - 4(11)(-13)}}{2(11)}$$

$$y = 0.581 \quad \text{or} \quad y = -2.035$$

$$\begin{aligned}x &= \frac{2(0.581) + 8}{3} & x &= \frac{2(-2.035) + 8}{3} \\ & & &= 1.31\end{aligned}$$

$$= 3.05 \quad \underline{\underline{x = 3.05, y = 0.581 \quad \text{or} \quad x = 1.31, y = -2.04}}$$

(Total for Question 21 is 5 marks)

22

A solid cube has a length of 4.8 cm, correct to 1 decimal place.
The cube has a mass of 220 grams, correct to 2 significant figures.

Work out the upper bound for the density of the cube.
Give your answer in g/cm^3 correct to 2 decimal places.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{upper bound density} = \frac{\text{upper mass}}{\text{lower volume}}$$

$$\begin{aligned} \text{lower volume} &= (\text{lower length})^3 \\ &= (4.75)^3 \\ &= 107.171875 \end{aligned}$$

$$\text{upper mass} = 225 \text{ g}$$

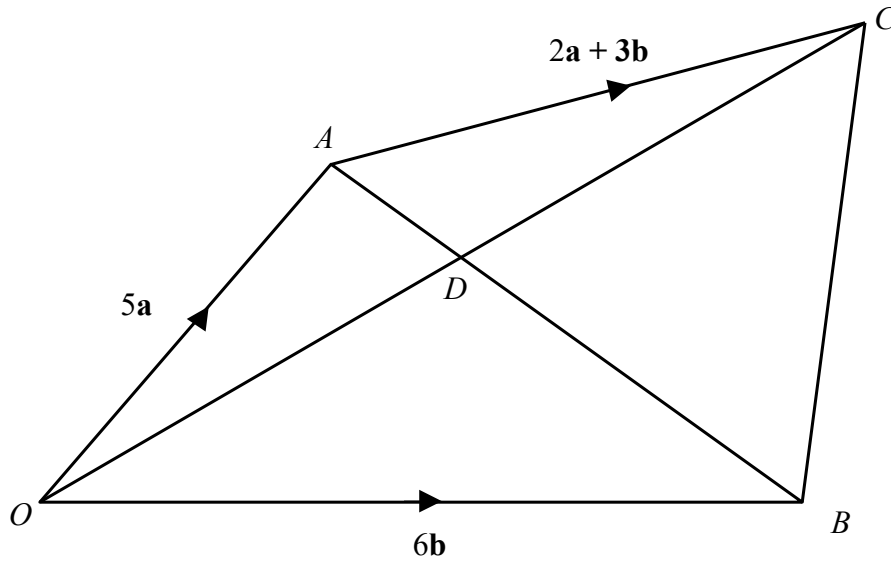
$$\frac{225}{107.17\dots}$$

$$\underline{2.10 \text{ g/cm}^3}$$

$$\underline{\hspace{1.5cm}} 2.10 \text{ g/cm}^3$$

(Total for Question 22 is 4 marks)

23 The diagram shows quadrilateral $OABC$.



ADB and ODC are straight lines.

$$\vec{OA} = 5\mathbf{a} \quad \vec{OB} = 6\mathbf{b} \quad \vec{AC} = 2\mathbf{a} + 3\mathbf{b}$$

Using a vector method, find the ratio $AD : DB$

$$\vec{AB} = -5\mathbf{a} + 6\mathbf{b}$$

$$\vec{AD} = x(-5\mathbf{a} + 6\mathbf{b})$$

$$\begin{aligned} \vec{OC} &= 5\mathbf{a} + 2\mathbf{a} + 3\mathbf{b} \\ &= 7\mathbf{a} + 3\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{AD} &= \vec{AO} + k(\vec{OC}) \\ &= -5\mathbf{a} + k(7\mathbf{a} + 3\mathbf{b}) \\ &= -5\mathbf{a} + 7k\mathbf{a} + 3k\mathbf{b} \end{aligned}$$

$$-5xa + 6xb = -5a + 7ka + 3kb$$

$$\begin{aligned} a \quad -5x &= -5 + 7k \end{aligned}$$

$$-5x = -5 + 7(2x)$$

$$-5x = -5 + 14x$$

$$5 = 19x$$

$$x = \frac{5}{19}$$

$$b \quad 6x = 3k$$

$$k = 2x$$

$$5 : 14$$

(Total for Question 23 is 5 marks)

$$\frac{5}{19} : \frac{14}{19} \quad \underline{\underline{5 : 14}}$$

TOTAL FOR PAPER IS 80 MARKS