# Mathematics <br> Paper 1 (Non-Calculator) Higher Tier 

## Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided
- there may be more space than you need.
- Calculators may not be used.

- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must show all your working out.


## Information

- The total mark for this paper is 80
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.


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## Higher Tier Formulae Sheet

## Perimeter, area and volume

Where $a$ and $b$ are the lengths of the parallel sides and $h$ is their perpendicular separation:

$$
\text { Area of a trapezium }=\frac{1}{2}(a+b) h
$$

Volume of a prism $=$ area of cross section $\times$ length Where $r$ is the radius and $d$ is the diameter:

Circumference of a circle $=2 \pi \mathrm{r}=\pi d$
Area of a circle $=\pi r^{2}$

## Pythagoras' Theorem and Trigonometry


b


In any right-angled triangle where $a, \mathrm{~b}$ and $c$ are the length of the sides and c is the hypotenuse:

$$
a^{2}+b^{2}=c^{2}
$$

In any right-angled triangle $A B C$ where $a, b$ and $c$ are the length of the sides and $c$ is the hypotenuse:

$$
\sin A=\frac{a}{c} \quad \cos A=\frac{b}{c} \quad \tan A=\frac{a}{b}
$$

In any triangle ABC where $\mathrm{a}, \mathrm{b}$ and c are the length of the sides:
sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule: $a^{2}=b^{2}+c^{2}-2 b c \cos A$
Area of triangle $=\frac{1}{2} a b \sin C$

## Probability

Where $\mathrm{P}(A)$ is the probability of outcome $A$ and $\mathrm{P}(B)$ is the probability of outcome $B$ :

$$
\begin{aligned}
& \mathrm{P}(A \text { or } B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \text { and } B) \\
& \mathrm{P}(A \text { and } B)=\mathrm{P}(A \text { given } B) \mathrm{P}(B)
\end{aligned}
$$

1 Write 204 as a product of its prime factors.

2 Show that $1 \frac{2}{3} \times 3 \frac{1}{5}=5 \frac{1}{3}$

3 Abbie is 9 years older than Ben.
Charlotte is twice as old as Abbie.
The sum of their three ages is 67
Find the ratio of Abbie's age to Bens's age to Charlotte's age

4 A shop sells packs of black pens, packs of red pens and packs of green pens.
There are
5 pens in each pack of black pens
4 pens in each pack of red pens
3 pens in each pack of green pens
On Monday,
$\begin{aligned} & \text { number of packs } \\ & \text { of black pens sold }\end{aligned}: \begin{aligned} & \begin{array}{l}\text { number of packs } \\ \text { of red pens sold }\end{array}\end{aligned}: \begin{aligned} & \text { number of packs } \\ & \text { of green pens sold }\end{aligned} \quad=8: 5: 2$
A total of 264 pens were sold.
Work out the number of green pens sold.
$5 \quad A D C$ is a triangle.

$A E D$ and $A B C$ are straight lines.
$E B$ is parallel to $D C$.
Angle $E B C=123^{\circ}$
Angle $A D C=56^{\circ}$
Work out the size of angle $E A B$.
You must give a reason for each stage of your working.

6 A car travels for 42 minutes at an average speed of $90 \mathrm{~km} / \mathrm{h}$.
(a) How far will the car travel in these 42 minutes?
$\qquad$ km

David says,
" 90 kilometres per hour is faster than 25 metres per second."
(b) Is David correct?

You must show how you get your answer

7 The table shows some information about the profit made each day at a cricket club on 100 days.

| Profit (fx) | Frequency |
| :---: | :---: |
| $0 \leqslant x<50$ | 8 |
| $50 \leqslant x<100$ | 12 |
| $100 \leqslant x<150$ | 20 |
| $150 \leqslant x<200$ | 30 |
| $200 \leqslant x<250$ | 19 |
| $250 \leqslant x<300$ | 11 |

(a) Complete the cumulative frequency table.

| Profit (£ $\boldsymbol{x}$ ) | Cumulative <br> Frequency |
| :---: | :---: |
| $0 \leqslant x<50$ |  |
| $0 \leqslant x<100$ |  |
| $0 \leqslant x<150$ |  |
| $0 \leqslant x<200$ |  |
| $0 \leqslant x<250$ |  |
| $0 \leqslant x<300$ |  |

(b) On the grid, draw a cumulative frequency graph for this information.

(2)
(c) Use your graph to find an estimate for the number of days the profit was less than $£ 160$
$\qquad$ days
(1)
(d) Use your graph to find an estimate for the interquartile range.
$\qquad$

8 (a) Write down the value of $27^{\circ}$
(b) Find the value of $3 \times 3^{4} \times 3^{-5}$
(c) Find the value of $5^{-3}$
(d) Find the value of $64^{\frac{1}{3}}$
$9 \quad$ Given that $\frac{a}{b}=\frac{3}{4} \quad$ and $\quad \frac{b}{c}=\frac{3}{7}$
Find $a: b: c$

10 Express $0.2 \dot{6} \dot{5}$ as a fraction.
You must show all your working.

11 The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.


The height of the cone is 12 cm .
The base of the cone has a diameter of 6 cm .
The hemisphere has a diameter of 6 cm .
The total volume of the shape is $k \pi \mathrm{~cm}^{3}$, where $k$ is an integer.
Work out the value of $k$.
$\qquad$

12 There are 8 counters in a bag.
5 of the counters are green.
3 of the counters are blue.
Riley takes at random two counters from the bag.
Work out the probability that Riley takes one counter of each colour.
You must show your working.

13 Prove that the sum of the squares of two consecutive odd numbers is always 2 more than a multiple of 8

14 Show that $\frac{8+\sqrt{18}}{3+\sqrt{2}}$ can be written in the form $\frac{a+\sqrt{2}}{b}$, where $a$ and $b$ are integers.

15 f and g are functions such that

$$
\mathrm{f}(x)=\frac{20}{\sqrt{x}} \quad \text { and } \quad \mathrm{g}(x)=4(3 x+2)
$$

(a) Find g(3)
(b) Find $\operatorname{gf}(16)$
(c) Find $\mathrm{g}^{-1}(2)$

16 The graph of $y=\mathrm{f}(x)$ is shown on the grid below.

(a) On the grid above, sketch the graph of $y=\mathrm{f}(x+2)$


On the grid, graph $\mathbf{A}$ has been reflected to give graph $\mathbf{B}$.
The equation of graph $\mathbf{A}$ is $y=\mathrm{g}(x)$
(b) Write down the equation of graph $\mathbf{B}$.

17 The point $P$ has coordinates $(5,2)$
The point $Q$ has coordinates $(a, b)$
A line perpendicular to $P Q$ is given by the equation $2 x-3 y=10$
Find an expression for $b$ in terms of $a$.
$18 x$ is proportional to $\sqrt{y}$ where $y>0$
$y$ is increased by $21 \%$
Work out the percentage increase in $x$.
$\qquad$

19 Given that $x^{2}-8 x+5=(x-a)^{2}-b$ for all values of $x$,
(i) find the value of a and the value of $b$.
$a=$ $\qquad$
$b=$ $\qquad$
(2)
(ii) Hence write down the coordinates of the turning point on the graph of $y=x^{2}-8 x+5$
( $\qquad$ , $\qquad$ )
$20 \quad C D E F$ is a parallelogram.

$M$ is the midpoint of $C E$.
$\overrightarrow{C D}=\mathbf{a}$ and $\overrightarrow{C F}=\mathbf{b}$
Use a vector method to prove that $M$ is the midpoint of the line $D F$.

21 Solve $\frac{3}{2 x-1}+\frac{2}{x+1}=1$
Give your answer in the form $\frac{p \pm \sqrt{q}}{2}$ where $p$ and $q$ are integers.

