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Write your name here

Surname

Other names

Centre Number

Candidate Number

Pearson Edexcel
Level 1/Level 2 GCSE (9–1)

Mathematics

Paper 3 (Calculator)

Higher Tier

Wednesday 8 November 2017 – Morning

Time: 1 hour 30 minutes

Paper Reference

1MA1/3H

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

• Use black ink or ball-point pen.
• Fill in the boxes at the top of this page with your name, centre number and candidate number.
• Answer all questions.
• Answer the questions in the spaces provided
  – there may be more space than you need.
• You must show all your working.
• Diagrams are NOT accurately drawn, unless otherwise indicated.
• Calculators may be used.
• If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.

Information

• The total mark for this paper is 80
• The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end.

Turn over

P49384A
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6/6/6/7/2/21
Answer ALL questions.
Write your answers in the spaces provided.
You must write down all the stages in your working.

1 The table shows information about the heights of 80 children.

<table>
<thead>
<tr>
<th>Height ($h$ cm)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$130 &lt; h \leq 140$</td>
<td>4</td>
</tr>
<tr>
<td>$140 &lt; h \leq 150$</td>
<td>11</td>
</tr>
<tr>
<td>$150 &lt; h \leq 160$</td>
<td>24</td>
</tr>
<tr>
<td>$160 &lt; h \leq 170$</td>
<td>22</td>
</tr>
<tr>
<td>$170 &lt; h \leq 180$</td>
<td>19</td>
</tr>
</tbody>
</table>

(a) Find the class interval that contains the median.

$$160 < h \leq 170$$

(1)

(b) Draw a frequency polygon for the information in the table.

(Total for Question 1 is 3 marks)
2 In London, 1 litre of petrol costs 108.9p
In New York, 1 US gallon of petrol costs $2.83

1 US gallon = 3.785 litres
£1 = $1.46

In which city is petrol better value for money, London or New York?
You must show your working.

\[
108.9 \times 3.785 = 412.1865 \text{ p per US gallon}
\]
\[
412.1865 \times 1.46 = 601.79229 \text{ cents per US gallon}
\]
\[
= 6.02
\]

New York is better value for money

(Total for Question 2 is 3 marks)

3 A gold bar has a mass of 12.5 kg.

The density of gold is 19.3 g/cm³

Work out the volume of the gold bar.
Give your answer correct to 3 significant figures.

\[
\text{Volume} = \frac{\text{Mass}}{\text{Density}}
\]
\[
= \frac{12500}{19.3}
= 648 \text{ cm}^3
\]

(Total for Question 3 is 3 marks)
4 There are only blue pens, green pens and red pens in a box.

The ratio of the number of blue pens to the number of green pens is 2 : 5
The ratio of the number of green pens to the number of red pens is 4 : 1

There are less than 100 pens in the box.

What is the greatest possible number of red pens in the box?

\[ \frac{B}{G} = \frac{2}{5} \times 4 \]
\[ \frac{G}{R} = \frac{1}{4} \times 5 \]

\[ B : G : R = 8 : 20 : 5 \]

\[ 8 : 20 : 5 \]

\[ 24 : 60 : 15 \]

\[ 33 \text{ parts} \times 3 = 99 \text{ parts} \]

\[ 15 \]

(Total for Question 4 is 3 marks)

5 (a) Find the value of the reciprocal of 1.6
Give your answer as a decimal.

\[ \frac{1}{1.6} \]

\[ 0.625 \]

(Jess rounds a number, \( x \), to one decimal place.
The result is 9.8)

(b) Write down the error interval for \( x \).

\[ 9.75 \leq x < 9.85 \]

(Total for Question 5 is 3 marks)
Here is a rectangle.

The length of the rectangle is \(7\) cm longer than the width of the rectangle.

4 of these rectangles are used to make this 8-sided shape.

The perimeter of the 8-sided shape is \(70\) cm.

Work out the area of the 8-sided shape.

\[
x + x + x + x + x + x + 7 + x + 7 = 8x + 42 = 70
\]

\[
8x = 28
\]

\[
x = 3.5
\]

\[
10.5 \times 3.5 = 36.75
\]

\[
4 \times 36.75 = 147 \text{ cm}^2
\]

\[
(\text{Total for Question 6 is 5 marks})
\]

\[
147 \text{ cm}^2
\]
Work out \((13.8 \times 10^7) \times (5.4 \times 10^{-12})\)
Give your answer as an ordinary number.

Type into the calculator.

\[
7.452 \times 10^{-4}
\]

\[
0.0007452
\]

(Total for Question 7 is 2 marks)
8 When a drawing pin is dropped it can land point down or point up.

Lucy, Mel and Tom each dropped the drawing pin a number of times.

The table shows the number of times the drawing pin landed point down and the number of times the drawing pin landed point up for each person.

<table>
<thead>
<tr>
<th></th>
<th>Lucy</th>
<th>Mel</th>
<th>Tom</th>
</tr>
</thead>
<tbody>
<tr>
<td>point down</td>
<td>31</td>
<td>53</td>
<td>16</td>
</tr>
<tr>
<td>point up</td>
<td>14</td>
<td>27</td>
<td>9</td>
</tr>
</tbody>
</table>

Rachael is going to drop the drawing pin once.

(a) Whose results will give the best estimate for the probability that the drawing pin will land point up?
   Give a reason for your answer.
   
   **Mel because she did the most trials.**  

(b) Use all the results in the table to work out an estimate for the probability that the drawing pin will land point up the first time and point down the second time.

\[ \text{Point up } \frac{1}{3} \quad \text{Point down } \frac{2}{3} \]

\[ \frac{1}{3} \times \frac{2}{3} = \frac{2}{9} \]

\[ \frac{2}{9} \]

(Total for Question 8 is 3 marks)
9 Jack bought a new boat for £12 500

The value, \( V \), of Jack's boat at the end of \( n \) years is given by the formula

\[
V = 12500 \times (0.85)^n
\]

(a) At the end of how many years was the value of Jack's boat first less than 50% of the value of the boat when it was new?

\[
6250 = 12500 \times 0.85^n
\]

\[
0.5 = 0.85^n
\]

\[
50\% = £6250
\]

\[
\quad n = 2 \quad V = £19031.25
\]

\[
\quad n = 4 \quad V = £6525.08
\]

\[
\quad n = 5 \quad V = £5546.32
\]

A savings account pays interest at a rate of \( R\% \) per year. Jack invests £5500 in the account for one year.

At the end of the year, Jack pays tax on the interest at a rate of 40%. After paying tax, he gets £79.20

(b) Work out the value of \( R \).

\[
£79.20 = 60\% \quad \div 6
\]

\[
£13.20 = 10\% \quad \div 10
\]

\[
£132 \quad \text{interest}
\]

\[
\frac{132}{5500} \times 100 = 2.4\%
\]

(Total for Question 9 is 5 marks)
10 There are only blue counters, yellow counters, green counters and red counters in a bag. A counter is taken at random from the bag.

The table shows the probabilities of getting a blue counter or a yellow counter or a green counter.

<table>
<thead>
<tr>
<th>Colour</th>
<th>Blue</th>
<th>Yellow</th>
<th>Green</th>
<th>Red</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.2</td>
<td>0.35</td>
<td>0.4</td>
<td>0.05</td>
</tr>
</tbody>
</table>

(a) Work out the probability of getting a red counter.

\[
1 - 0.4 - 0.35 - 0.2 = 0.05
\]

(b) What is the least possible number of counters in the bag?
You must give a reason for your answer.

\[
\begin{align*}
20 &
35 &
40 &
5 \\
4 &
7 &
8 &
1
\end{align*}
\]

\[\text{HCF} = 5\]

The least number of counters in the bag is 20.

You cannot divide the numbers any further and get whole number answers.

(Total for Question 10 is 3 marks)
The cumulative frequency graph shows information about the weights of 60 potatoes.

(a) Use the graph to find an estimate for the median weight.

Jamil says,

“80 - 40 = 40 so the range of the weights is 40g.”

(b) Is Jamil correct?
   You must give a reason for your answer.

Possibly not because the data would have been grouped so the actual highest may not be 80 and the actual lowest may not be 40.
(c) Show that less than 25% of the potatoes have a weight greater than 65 g.

\[
\frac{11.5}{60} \times 100 = 19.17\% \\
\]
This is less than 25%.

(Total for Question 11 is 4 marks)
Alan has two spinners, spinner A and spinner B. Each spinner can land on only red or white.

The probability that spinner A will land on red is 0.25
The probability that spinner B will land on red is 0.6

The probability tree diagram shows this information.

Alan spins spinner A once and he spins spinner B once. He does this a number of times.

The number of times both spinners land on red is 24

Work out an estimate for the number of times both spinners land on white.

\[
\begin{align*}
\text{Red} & \quad \text{Red} \\
0.25 \times 0.6 & = 0.15 & \quad [\text{24 times}] \\
\text{white} & \quad \text{white} \\
0.75 \times 0.4 & = 0.3 & \quad [\text{48 times}] \\
\text{[0.3 is 2 x 0.15]} & \\
\end{align*}
\]

(Total for Question 12 is 3 marks)
13 Write $x^2 + 6x - 7$ in the form $(x + a)^2 + b$ where $a$ and $b$ are integers.

$$(x + 3)^2 - 9 - 7$$

$$(x + 3)^2 - 16$$

(Total for Question 13 is 2 marks)

14 Cone A and cone B are mathematically similar.

The ratio of the volume of cone A to the volume of cone B is $27 : 8$

The surface area of cone A is 297 cm$^2$

Show that the surface area of cone B is 132 cm$^2$

\[
\begin{array}{ccc}
\text{Volume} & \text{S.F} & 27 : 8 \\
\text{Length} & \text{S.F} & 3 : 2 \\
\text{Area} & \text{S.F} & 9 : 4
\end{array}
\]

\[
\frac{297}{9} \times 4 = 132 \text{ cm}^2
\]

(Total for Question 14 is 3 marks)
15 (a) Show that the equation \( x^3 + 7x - 5 = 0 \) has a solution between \( x = 0 \) and \( x = 1 \)

\[
(0)^3 + 7(0) - 5 = -5 \\
(1)^3 + 7(1) - 5 = 3
\]

change of sign: there is a solution between 0 and 1

(b) Show that the equation \( x^3 + 7x - 5 = 0 \) can be arranged to give \( x = \frac{5}{x^2 + 7} \)

\[
x^3 + 7x = 5 \\
x(x^2 + 7) = 5 \\
x = \frac{5}{x^2 + 7}
\]

(c) Starting with \( x_0 = 1 \), use the iteration formula \( x_{n+1} = \frac{5}{x_n^2 + 7} \) three times to find an estimate for the solution of \( x^3 + 7x - 5 = 0 \)

\[
x_1 = \frac{5}{(1)^2 + 7} = 0.625 \\
x_2 = \frac{5}{(0.625)^2 + 7} = 0.6765327696 \\
x_3 = 0.6704483001
\]

\[0.670\overline{3}(3sf)\]
(d) By substituting your answer to part (c) into \( x^3 + 7x - 5 \),

comment on the accuracy of your estimate for the solution to \( x^3 + 7x - 5 = 0 \)

\[
\text{Ans}^3 + 7(\text{Ans}) - 5 = -5.49 \times 10^{-3}
\]

\[
= -0.00549
\]

The answer is very close to 0, so an accurate estimate.

(Total for Question 15 is 9 marks)

16 The petrol consumption of a car, in litres per 100 kilometres, is given by the formula

\[
\text{Petrol consumption} = \frac{100 \times \text{Number of litres of petrol used}}{\text{Number of kilometres travelled}}
\]

Nathan’s car travelled 148 kilometres, correct to 3 significant figures. The car used 11.8 litres of petrol, correct to 3 significant figures.

Nathan says,

“My car used less than 8 litres of petrol per 100 kilometres.”

Could Nathan be wrong?
You must show how you get your answer.

\[
\text{Upper petrol consumption} = \frac{100 \times \text{Upper litres}}{\text{Lower distance}}
\]

\[
11.7 \uparrow 11.8 \uparrow 11.9
\]

\[
11.75 \quad 11.85
\]

\[
\text{Upper petrol consumption} = \frac{100 \times 11.85}{147.5}
\]

\[
= 8.03 \quad (3\text{sf})
\]

Nathan could be wrong.

(Total for Question 16 is 3 marks)
17 \( ABC \) and \( ADC \) are triangles.

The area of triangle \( ADC \) is 56 \( m^2 \)

Work out the length of \( AB \).
Give your answer correct to 1 decimal place.

\[
\begin{align*}
\text{\( ADC \):} & \quad \frac{1}{2} \, ab \sin C = 56 \\
& \quad \frac{1}{2} \, (11 \times x) \sin(105) = 56 \\
& \quad x = \frac{56}{\frac{1}{2} \, (11) \sin(105)} \\
& \quad = 10.54099384
\end{align*}
\]

\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc \cos A \\
y^2 &= 11^2 + \text{Ans}^2 - 2(11)(\text{Ans}) \cos(105) \\
y^2 &= 292.1331702 \\
y &= 17.09190364
\end{align*}
\]

\[
\begin{align*}
\frac{\alpha}{\sin A} &= \frac{b}{\sin B} \\
\frac{\alpha}{\sin 48} &= \frac{\text{Ans}}{\sin 118} \\
\alpha &= \frac{\text{Ans}}{\sin 118} \times \sin 48 \\
&= 14.4 \quad (\text{1dp})
\end{align*}
\]

(Total for Question 17 is 5 marks)
18 Here is a speed-time graph for a train.

(a) Work out an estimate for the distance the train travelled in the first 20 seconds.
Use 4 strips of equal width.

\[ \frac{2 \times 5}{2} + \frac{2+5}{2} \times 5 + \frac{5+10}{2} \times 5 + \frac{10+18}{2} \times 5 \]

\[ \underline{130} \text{ m} \]

(b) Is your answer to (a) an underestimate or an overestimate of the actual distance the train travelled?
Give a reason for your answer.

\underline{Overestimate. The area of the trapeziums is greater than the area under the curve.} 

(Total for Question 18 is 4 marks)
Prove algebraically that the straight line with equation \( x - 2y = 10 \) is a tangent to the circle with equation \( x^2 + y^2 = 20 \)

Tangent will mean there is one solution.

\[
\begin{align*}
\ x & = (10 + 2y) \\
(10 + 2y) + y^2 & = 20 \\
(10 + 2y)(10 + 2y) + y^2 & = 20 \\
100 + 20y + 20y + 4y^2 + y^2 & = 20 \\
5y^2 + 40y + 100 & = 20 \\
5y^2 + 40y + 80 & = 0 \\
y^2 + 8y + 16 & = 0 \\
(y + 4)(y + 4) & = 0 \\
y & = -4 \\
x & = 10 + 2(-4) \\
x & = 2
\end{align*}
\]
A, B and C are points on the circumference of a circle, centre O. AOB is a diameter of the circle.

Prove that angle ACB is 90°
You must not use any circle theorems in your proof.

Let \( \hat{AOC} = x \)
\( \hat{OAC} = \hat{OCA} = \frac{180 - x}{2} = 90 - \frac{1}{2}x \) (Angles at the base of an isosceles triangle are equal)

Let \( \hat{BOC} = y \)
\( \hat{OBC} = \hat{OCB} = \frac{180 - y}{2} = 90 - \frac{1}{2}y \)

\[ \hat{ACB} = 90 - \frac{1}{2}x + 90 - \frac{1}{2}y \]
\[ = 180 - \left( \frac{1}{2}x + \frac{1}{2}y \right) \]
\[ = 180 - \left( \frac{1}{2}x + \frac{1}{2}y \right) \]
\[ = 180 = x + y \] (Angle on a straight line add up to 180°)
\[ \therefore 90 = \frac{1}{2}x + \frac{1}{2}y \]
\[ \hat{ACB} = 180 - 90 \]
\[ = 90 \]

(Total for Question 20 is 4 marks)
OAN, OMB and APB are straight lines.
AN = 2OA.
M is the midpoint of OB.
\[ \overrightarrow{OA} = a \quad \overrightarrow{OB} = b \]
\[ \overrightarrow{AP} = k \overrightarrow{AB} \] where \( k \) is a scalar quantity.

Given that MPN is a straight line, find the value of \( k \).
\[ \overrightarrow{AB} = -a + b \]
\[ \overrightarrow{NM} = -3a + \frac{1}{2}b \]
\[ \overrightarrow{AP} = k(-a + b) \]
\[ \overrightarrow{NP} = -2a + k(-a + b) \]

\[ MPN \text{ is a straight line} \Rightarrow \overrightarrow{NP} \times x = \overrightarrow{NM} \]
\[ x(-2a + k(-a + b)) = -3a + \frac{1}{2}b \]
\[ x(-2a - ka + kb) = -3a + \frac{1}{2}b \]
\[ -2xa - kxa + kxb = -3a + \frac{1}{2}b \]

Split \( a \) and \( b \)

Parallel
\[ -2x - kx = -3 \quad kx = \frac{1}{2} \]
\[ 2x + kx = 3 \quad x = \frac{1}{2k} \]
\[ 2\left(\frac{1}{2k}\right) + k\left(\frac{1}{2k}\right) = 3 \]
\[ \frac{1}{k} + \frac{1}{2} = 3 \]
\[ \frac{1}{k} = \frac{5}{2} \quad k = \frac{2}{5} \]

(Total for Question 21 is 5 marks)