Mathematics
Paper 1 (Non-Calculator)

Higher Tier

Thursday 2 November 2017 – Morning
Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.
Tracing paper may be used.

Instructions

● Use black ink or ball-point pen.
● Fill in the boxes at the top of this page with your name, centre number and candidate number.
● Answer all questions.
● Answer the questions in the spaces provided – there may be more space than you need.
● You must show all your working.
● Diagrams are NOT accurately drawn, unless otherwise indicated.
● Calculators may not be used.

Information

● The total mark for this paper is 80
● The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

● Read each question carefully before you start to answer it.
● Keep an eye on the time.
● Try to answer every question.
● Check your answers if you have time at the end.
Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. Write 36 as a product of its prime factors.

\[
\begin{align*}
36 &= 2 \times 18 \\
   &= 2 \times 2 \times 9 \\
   &= 2 \times 2 \times 3 \times 3 \\
\text{or} & \quad 2^2 \times 3^2
\end{align*}
\]

(Total for Question 1 is 2 marks)
2. Kiaria is 7 years older than Jay. Martha is twice as old as Kiaria. The sum of their three ages is 77.

Find the ratio of Jay’s age to Kiaria’s age to Martha’s age.

\[
\begin{align*}
K &= J + 7 \quad \rightarrow \quad (J) = (K - 7) \\
(2K) &= (M)
\end{align*}
\]

\[
\begin{align*}
K + (J) + (M) &= 77 \\
K + (K - 7) + (2K) &= 77 \\
k + k - 7 + 2k &= 77 \\
4k - 7 &= 77 \\
4k &= 84 \\
k &= 21
\end{align*}
\]

\[
\begin{align*}
J &= 21 - 7 = 14 \\
M &= 2k \\
&= 2(21) = 42
\end{align*}
\]

\[
\begin{align*}
J : k : M \\
14 : 21 : 42 \\
2 : 3 : 6
\end{align*}
\]

(Total for Question 2 is 4 marks)
$ABCD$ is a parallelogram.
$EDC$ is a straight line.
$F$ is the point on $AD$ so that $BFE$ is a straight line.

Angle $EFD = 35^\circ$
Angle $DCB = 75^\circ$

Show that angle $ABF = 70^\circ$
Give a reason for each stage of your working.

$\hat{AFB} = 35^\circ$ Vertically opposite angles are equal.
$\hat{BAF} = 75^\circ$ Opposite angles in a parallelogram are equal.

$\hat{ABF} = 180 - 35 - 75 = 70^\circ$ Angles in a triangle sum to $180^\circ$
4 The diagram shows a logo made from three circles.

Each circle has centre $O$.

Daisy says that exactly $\frac{1}{3}$ of the logo is shaded.

Is Daisy correct?
You must show all your working.

**Big Circle**
Area = $\pi (10)^2$
= $100\pi$

**Medium Circle**
Area = $\pi (7)^2$
= $49\pi$

**Small Circle**
Area = $\pi (4)^2$
= $16\pi$

**Shaded Area**
$49\pi - 16\pi = 33\pi$

**Total Area**
$100\pi$

$\frac{33\pi}{100\pi} = \frac{33}{100}\text{ shaded.} = 33\%$

This is not $\frac{1}{3}$, $\frac{1}{3}$ is 33.3%.

(Total for Question 4 is 4 marks)
5 The table shows information about the weekly earnings of 20 people who work in a shop.

<table>
<thead>
<tr>
<th>Weekly earnings (£x)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$150 &lt; x \leq 250$</td>
<td>200</td>
</tr>
<tr>
<td>$250 &lt; x \leq 350$</td>
<td>300</td>
</tr>
<tr>
<td>$350 &lt; x \leq 450$</td>
<td>400</td>
</tr>
<tr>
<td>$450 &lt; x \leq 550$</td>
<td>500</td>
</tr>
<tr>
<td>$550 &lt; x \leq 650$</td>
<td>600</td>
</tr>
</tbody>
</table>

(a) Work out an estimate for the mean of the weekly earnings.

\[
\frac{73000}{200} = £365
\]

Nadiya says,

“The mean may not be the best average to use to represent this information.”

(b) Do you agree with Nadiya?
You must justify your answer.

Yes. The mean is affected by extreme values. The median may be better.

(Total for Question 5 is 4 marks)
6 Here is a rectangle.

All measurements are in centimetres.
The area of the rectangle is 48 cm².
Show that \( y = 3 \)

\[
\begin{align*}
2x + 6 &= 5x - 9 \\
-2x &= -2x \\
6 &= 3x - 9 \\
+9 &= +9 \\
15 &= 3x \\
x &= 5 \\
2(5) + 6 &= 16 \\
16y &= 48 \\
y &= 3
\end{align*}
\]

(Total for Question 6 is 4 marks)
Brogan needs to draw the graph of \( y = x^2 + 1 \)

Here is her graph.

Write down one thing that is wrong with Brogan’s graph.

Brogan should have drawn a curve

(Total for Question 7 is 1 mark)
8 Write these numbers in order of size.  
Start with the smallest number.

\[
0.246 \quad 0.246 \quad 0.246 \quad 0.246 \\
0.24646 \quad 0.24666 \quad 0.246246 \quad 0.246
\]

\[
0.246, \quad 0.246, \quad 0.246, \quad 0.246
\]

(Total for Question 8 is 2 marks)

9 James and Peter cycled along the same 50 km route. 
James took \( 2 \frac{1}{2} \) hours to cycle the 50 km.

Peter started to cycle 5 minutes after James started to cycle.  
Peter caught up with James when they had both cycled 15 km.

James and Peter both cycled at constant speeds. 

Work out Peter’s speed.

\[
\text{James’ speed} = \frac{50}{2.5} = 20 \text{ km/hour}
\]

\[
\text{For 15 km, James} \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{15}{20} \\
= \frac{3}{4} = 45 \text{ mins.}
\]

Peter. It took Peter \( \frac{40}{60} \) mins to cycle 15 km.  
\[
\text{speed} = \frac{\text{distance}}{\text{time}} \\
= \frac{15}{\frac{2}{3}} = 22.5 \text{ km/h}
\]

(Total for Question 9 is 5 marks)
(a) Write down the value of $100^{\frac{1}{2}}$

(b) Find the value of $125^{\frac{2}{3}}$

\[
125^{\frac{2}{3}} = 5^2 = 25
\]

(Total for Question 10 is 3 marks)

11 3 teas and 2 coffees have a total cost of £7.80
5 teas and 4 coffees have a total cost of £14.20

Work out the cost of one tea and the cost of one coffee.

\[
\begin{align*}
3t + 2c &= 7.8 \\
5t + 4c &= 14.2
\end{align*}
\]

\[
\begin{align*}
15t + 10c &= 39 \\
15t + 12c &= 42.6
\end{align*}
\]

\[
\begin{align*}
2c &= 3.6 \\
c &= 1.8 \quad [\text{£1.80}]
\end{align*}
\]

\[
\begin{align*}
3t + 2(1.8) &= 7.8 \\
3t + 3.6 &= 7.8
\end{align*}
\]

\[
\begin{align*}
3t &= 4.2 \\
t &= 1.4 \quad [\text{£1.40}]
\end{align*}
\]

(Total for Question 11 is 4 marks)
The table shows information about the heights, in cm, of a group of Year 11 girls.

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Least height</td>
<td>154</td>
</tr>
<tr>
<td>Median</td>
<td>165</td>
</tr>
<tr>
<td>Lower quartile</td>
<td>161</td>
</tr>
<tr>
<td>Interquartile range</td>
<td>7</td>
</tr>
<tr>
<td>Range</td>
<td>20</td>
</tr>
</tbody>
</table>

\[\text{Upper Quartile} = 168\]
\[\text{Highest} = 174\]

(a) Draw a box plot for this information.

The box plot below shows information about the heights, in cm, of a group of Year 7 girls.

(b) Compare the distribution of heights of the Year 7 girls with the distribution of heights of the Year 11 girls.

The median height of the year 11 girls is greater. \((\text{Higher Average})\)

The IQR of the year 11 girls is smaller. \((\text{Less Spread})\)

(Total for Question 12 is 5 marks)
A factory makes 450 pies every day. The pies are chicken pies or steak pies.

Each day Milo takes a sample of 15 pies to check.

The proportion of the pies in his sample that are chicken is the same as the proportion of the pies made that day that are chicken.

On Monday Milo calculated that he needed exactly 4 chicken pies in his sample.

(a) Work out the total number of chicken pies that were made on Monday.

\[
\frac{4}{15} \text{ chicken pies} \times 450 = 120
\]

On Tuesday, the number of steak pies Milo needs in his sample is 6, correct to the nearest whole number.

Milo takes at random a pie from the 450 pies made on Tuesday.

(b) Work out the lower bound of the probability that the pie is a steak pie.

\[
\frac{5.5}{15} \times 450 = \frac{11}{30}
\]

(Total for Question 13 is 4 marks)
14 The ratio \((y + x) : (y - x)\) is equivalent to \(k : 1\)

Show that \(y = \frac{x(k + 1)}{k - 1}\)

\[
\frac{y + x}{y - x} = \frac{k}{1}
\]

\[y + x = k(y - x)\]

\[y + x = ky - kx\]

\[y + x + kx = ky\]

\[x + kx = ky - y\]

\[xc(1+k) = y(k-1)\]

\[y = \frac{xc(1+k)}{k-1} = \frac{x(k+1)}{k-1}\]

(Total for Question 14 is 3 marks)

15 \(x = 0.4\overline{36}\)

Prove algebraically that \(x\) can be written as \(\frac{24}{55}\)

\[0.43\overline{6} = x\]

\[4.3\overline{6} = 10x\]

\[436.3\overline{6} = 1000x\]

\[432 = 990x\]

\[x = \frac{432}{990} = \frac{216}{495} = \frac{24}{55}\]

(Total for Question 15 is 3 marks)
16  $y$ is directly proportional to $\sqrt[3]{x}$

$y = 1 \frac{1}{6}$ when $x = 8$

Find the value of $y$ when $x = 64$

$$y = k \sqrt[3]{x}$$

$$\frac{7}{6} = k \sqrt[3]{8}$$

$$\frac{7}{6} = 2k$$

$$k = \frac{7}{12}$$

$$y = \frac{7}{12} \sqrt[3]{64}$$

$$y = \frac{7}{12} \cdot 4 = \frac{7}{3}$$

(Total for Question 16 is 3 marks)

17  $n$ is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n + 1)$ and $\frac{1}{2}(n + 1)(n + 2)$ is always a square number.

$$\frac{1}{2} n(n + 1) + \frac{1}{2} (n+1)(n+2)$$

$$\frac{1}{2} n^2 + \frac{1}{2} n + \frac{1}{2} (n^2 + 2n + n + 2)$$

$$\frac{1}{2} n^2 + \frac{1}{2} n + \frac{1}{2} (n^2 + 3n + 2)$$

$$\frac{1}{2} n^2 + \frac{1}{2} n + \frac{1}{2} n^2 + \frac{3}{2} n + 1$$

$$n^2 + 2n + 1$$

$$(n+1)(n+1)$$

$$(n+1)^2$$

(Total for Question 17 is 2 marks)
Enlarge shape $P$ by scale factor $\frac{1}{2}$ with centre of enlargement $(0, 0)$.
Label your image $Q$.

(Total for Question 18 is 2 marks)
ABCD is a rectangle.

A, E and B are points on the straight line L with equation $x + 2y = 12$

A and D are points on the straight line M.

$AE = EB$

Find an equation for M.

\[ L: \quad x + 2y = 12 \]
\[ 2y = -x + 12 \]
\[ y = -\frac{1}{2}x + 6 \]

\[ \text{Gradient of } L \quad \text{is} \quad -\frac{1}{2} \]

\[ L \text{ and } M \text{ are perpendicular} \]

\[ m = 2 \]

\[ y = 2x + C \]
\[ 12 = 2(-12) + C \]
\[ 12 = -24 + C \]
\[ C = 36 \]

(Crosses x when y = 0)

\[ x = 12 \]

(Total for Question 19 is 4 marks)
20 The table shows some values of \( x \) and \( y \) that satisfy the equation \( y = a \cos x^\circ + b \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3</td>
<td>1 + ( \sqrt{3} )</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1 - ( \sqrt{3} )</td>
<td>-1</td>
</tr>
</tbody>
</table>

Find the value of \( y \) when \( x = 45 \)

\[
\cos(0) = 1
\]
\[
\frac{3}{\alpha + b} = \cos(90) = 0
\]
\[
1 = b
\]
\[
\alpha = 2
\]

\[
y = 2 \cos(x) + 1
\]
\[
= 2 \cos(45) + 1
\]
\[
= 2 \left( \frac{\sqrt{2}}{2} \right) + 1
\]
\[
= \sqrt{2} + 1
\]

(Total for Question 20 is 4 marks)

21 Show that \( \frac{6 - \sqrt{8}}{\sqrt{2} - 1} \) can be written in the form \( a + b\sqrt{2} \) where \( a \) and \( b \) are integers.

\[
\sqrt{8} = \sqrt{4\sqrt{2}} = 2\sqrt{2}
\]
\[
\frac{(6 - 2\sqrt{2})(\sqrt{2}+1)}{(\sqrt{2} - 1)(\sqrt{2}+1)}
\]
\[
6\sqrt{2} + 6 - 4 - 2\sqrt{2}
\]
\[
= \frac{2 + \sqrt{2} - \sqrt{2} - 1}{4\sqrt{2} + 2}
\]
\[
= \frac{1}{4\sqrt{2} + 2}
\]

(Total for Question 21 is 3 marks)
The two triangles in the diagram are similar.

There are two possible values of $x$.

Work out each of these values.
State any assumptions you make in your working.

Either $AB = \text{S.F} \times AD$ or $AE = \text{S.F} \times AD$

\[
\frac{AE}{AD} = \text{scale factor}
\]

\[
\frac{15}{12} = \frac{5}{4}
\]

\[
8 \times \frac{5}{4} = 10 \text{ cm} \quad \therefore \quad x = 10 - 8 = 2 \text{ cm}
\]

\[
\frac{AB}{AD} = \text{scale factor}
\]

\[
\frac{15}{8} = 12 \times \frac{15}{8} = 22.5 \text{ cm}
\]

\[
\therefore \quad x = 22.5 - 8 = 14.5 \text{ cm}
\]

(Total for Question 22 is 5 marks)
23 Here is a rectangle and a right-angled triangle.

All measurements are in centimetres.
The area of the rectangle is greater than the area of the triangle.
Find the set of possible values of $x$.

\[(x - 1)(3x - 2) > x^2\]
\[3x^2 - 2x - 3x + 2 > x^2\]
\[2x^2 - 5x + 2 > 0\]
\[(2x - 1)(x - 2) > 0\]
\[x = \frac{1}{2} \quad x = 2\]
\[x < \frac{1}{2} \text{ and } x > 2\]

But $x$ cannot be less than $\frac{1}{2}$ because $(x - 1)$ would be negative

So $x > 2$

$\times > 2$

(Total for Question 23 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS