Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – **there may be more space than you need**.
- **Calculators must not be used.**

Information

- The total mark for this paper is 100
- The marks for **each** question are shown in brackets – **use this as a guide as to how much time to spend on each question**.
- Questions labelled with an **asterisk (**) are ones where the quality of your written communication will be assessed.**

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.
GCSE Mathematics 1MA0

Formulae: Higher Tier

You must not write on this formulae page.
 Anything you write on this formulae page will gain NO credit.

Volume of prism = area of cross section × length

Area of trapezium = \( \frac{1}{2} (a + b)h \)

Volume of sphere = \( \frac{4}{3} \pi r^3 \)
Surface area of sphere = \( 4\pi r^2 \)

Volume of cone = \( \frac{1}{3} \pi r^2 h \)
Curved surface area of cone = \( \pi rl \)

In any triangle \( ABC \)

Sine Rule \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

Cosine Rule \( a^2 = b^2 + c^2 - 2bc \cos A \)

Area of triangle = \( \frac{1}{2} ab \sin C \)

The Quadratic Equation
The solutions of \( ax^2 + bx + c = 0 \) where \( a \neq 0 \), are given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Answer ALL questions.

Write your answers in the spaces provided.

You must write down all stages in your working.

You must NOT use a calculator.

1. The diagram shows a prism.

![Diagram of a prism with a hexagonal cross-section and dimensions 30 cm^2 and 25 cm]

The area of the cross section of the prism is 30 cm^2.
The length of the prism is 25 cm.

Work out the volume of the prism.

\[
30 \times 25
\]

\[
\begin{array}{c|c}
30 & \\
20 & 600 \\
5 & 150 \\
\end{array}
\]

\[750 \text{ cm}^3\]

(Total for Question 1 is 3 marks)
(a) Reflect shape $P$ in the line $x = -1$
(b) Rotate shape $P$ $90^\circ$ anticlockwise about $(0, 1)$.

(Total for Question 2 is 4 marks)
3 Richard wants to find out how often people buy crisps. He uses this question on a questionnaire:

How often do you buy crisps?

Often ☐  Sometimes ☐  Never ☐

(a) Write down two things that are wrong with this question.

1. There is no timescale

2. It is not clear what ‘often’, ‘sometimes’, and ‘never’ mean.

(b) Design a better question for Richard to use on his questionnaire to find out how often people buy crisps.

How many times do you buy crisps a week?

☐ 0  ☐ 1-2  ☐ 3-4  ☐ More

Richard is going to ask the students in his maths class to answer his questionnaire.

(c) This may not be a good sample to use.

Give one reason why.

They will all be the same age.

(Total for Question 3 is 5 marks)
4 (a) Simplify $p^2 \times p^3$

(b) Simplify $g^6 \div g^4$

(c) Simplify $(k^3)^2$

$$k^3 \times k^3 = k^6$$

(d) Expand and simplify $3(m + 4) - 2(4m + 1)$

$$3m + 12 - 8m - 2 = -5m + 10$$

(e) Factorise $n^2 - 7n$

$$n(n - 7)$$

(Total for Question 4 is 6 marks)
5 There are 892 litres of oil in Mr Aston’s oil tank. He uses 18.7 litres of oil each day. Estimate the number of days it will take him to use all the oil in the tank.

\[ \frac{900}{20} \]

(Total for Question 5 is 2 marks)

6 One of the teachers at a school is chosen at random. The probability that this teacher is female is \( \frac{3}{5} \). There are 36 male teachers at the school. Work out the total number of teachers at the school.

\[ \frac{2}{5} \text{ are male} \]

\[ \frac{2}{5} = 36 \]

\[ \frac{1}{5} = 18 \]

\[ 18 \times 5 = 90 \]

(Total for Question 6 is 3 marks)
The diagram shows the plan of a floor.

Diagram NOT accurately drawn

Angie is going to varnish the floor.

She needs 1 litre of varnish for 5 m\(^2\) of floor.

There are 2.5 litres of varnish in each tin of varnish.

Angie has 3 tins of varnish.

Does she have enough varnish for all the floor?

You must show all your working.

\[ \text{Big Rectangle} \quad 6 \times 10 = 60 \text{ m}^2 \]

\[ \text{(L) Little Rectangle} \quad 2 \times 5 = 10 \text{ m}^2 \]

Floor Area = \( 60 - 10 = \frac{50 \text{ m}^2}{10} \times \]

\[ \frac{1 \text{ litre for } 5 \text{ m}^2}{10 \text{ litres for } 50 \text{ m}^2} \]

\[ 3 \times 2.5 = 7.5 \text{ litres} \]

She does not have enough, she needs 10 litres. She has 7.5 litres.

(Total for Question 7 is 5 marks)
8 Carol spins a spinner 80 times.

The table shows information about her results.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>39</td>
</tr>
<tr>
<td>K</td>
<td>25</td>
</tr>
<tr>
<td>L</td>
<td>16</td>
</tr>
</tbody>
</table>

Dan spins this spinner 300 times.

Work out an estimate for the number of times that Dan will get an L.

\[
\frac{16}{80} = \frac{8}{40} = \frac{4}{20} = \frac{2}{10} = \frac{1}{5}
\]

\[
\frac{1}{5} \text{ of } 300 = 60
\]

(Total for Question 8 is 3 marks)

9 A shop sells packets of envelopes.

There are 5 envelopes in a small packet.
There are 20 envelopes in a large packet.

There is a total of \( T \) envelopes in \( x \) small packets and \( y \) large packets.

Write down a formula for \( T \) in terms of \( x \) and \( y \).

\[
T = 5x + 20y
\]

(Total for Question 9 is 3 marks)
Diagram NOT accurately drawn

Point $P$ has coordinates $(5, 7)$.
Point $M$ has coordinates $(1, 2.5)$.
Point $M$ is the midpoint of the line $PQ$.
Find the coordinates of point $Q$.

$(-3, -2)$

(Total for Question 10 is 2 marks)
11 66 people went on a day trip.
Each person did only one activity on the trip.

Each person went skating or went to an art gallery or went bowling.

43 of the people are female.
4 of the 10 people who went skating are male.
20 of the people went to the art gallery.
10 males went bowling.

Work out the number of females who went to the art gallery.

<table>
<thead>
<tr>
<th></th>
<th>Skating</th>
<th>Art G.</th>
<th>Bowling</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>4</td>
<td>9</td>
<td>10</td>
<td>23</td>
</tr>
<tr>
<td>Female</td>
<td>6</td>
<td>11</td>
<td>26</td>
<td>43</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>20</td>
<td>36</td>
<td>66</td>
</tr>
</tbody>
</table>

(Total for Question 11 is 4 marks)
12 The diagram shows a circle inside a square.

![Diagram NOT accurately drawn]

\(ABCD\) is a square of side 10 cm.
Each side of the square is a tangent to the circle.

Work out the total area of the shaded regions in terms of \(\pi\).
Give your answer in its simplest form.

\[
\begin{align*}
\text{Area of Square} &= 10 \times 10 = 100 \text{ cm}^2 \\
\text{Area of Circle} &= \pi r^2 \\
&= \pi \times 5^2 \\
&= 25\pi \\
\text{Shaded Area} &= 100 - 25\pi
\end{align*}
\]

\(100 - 25\pi\) cm\(^2\)

(Total for Question 12 is 3 marks)
13 The table gives information about Ali's spending last month.

<table>
<thead>
<tr>
<th>Item</th>
<th>Percentage of total spending</th>
</tr>
</thead>
<tbody>
<tr>
<td>rent</td>
<td>30%</td>
</tr>
<tr>
<td>food</td>
<td>15%</td>
</tr>
<tr>
<td>transport</td>
<td>12%</td>
</tr>
<tr>
<td>other</td>
<td>43%</td>
</tr>
</tbody>
</table>

Ali's total spending last month was £800

Next month Ali's rent, in pounds, is going to rise by 20%.
His total spending will still be the same.

Express the amount of money Ali will spend on rent next month as a percentage of £800

\[
\text{Rent last month: } 30\% \text{ of } 800 = \frac{10\% \times 800}{30\%} = \frac{240}{80} = \frac{12}{4} = 36\%
\]

\[
\text{New rent } = 240 + 20\% \\
\quad 10\% \times 240 = 24 \\
\quad 20\% \times 240 = 48 \\
\quad = \frac{1288}{800} = \frac{144}{400} = \frac{72}{200} = \frac{36}{100} = 36\%
\]

(Total for Question 13 is 3 marks)
14 (a) Use ruler and compasses to bisect the angle at $A$.
You must show all your construction lines.

(b) Use ruler and compasses to construct the perpendicular from the point $P$ to the line $QR$.
You must show all your construction lines.

(Total for Question 14 is 4 marks)
The box plots give information about the wages of a group of 16 year old workers and a group of 18 year old workers.

*(a) Compare the distribution of the wages of the 16 year old workers with the distribution of the wages of the 18 year old workers.

The median wage of the 18 year olds is greater. (£240 compared to £180). On average they earn more.

The interquartile range of 18 year olds is bigger - their wages are more spread out.
There are 200 workers who are 16 years old.

(b) Work out an estimate for the number of these workers whose wages are £130 or more.

\[
\underbrace{130} \text{ is the lower quartile} \\
\frac{3}{4} \text{ above the lower quartile} \\
\frac{3}{4} \times 200 = 150
\]

(Total for Question 15 is 5 marks)

16 Work out the value of \( (3.5 \times 10^6) \div (5 \times 10^{-3}) \)
Give your answer in standard form.

\[
\frac{3.5 \times 10^6}{5 \times 10^{-3}} = \frac{7 \times 10^9}{10^9} = 7 \times 10^8
\]

(Total for Question 16 is 2 marks)
17 (a) Solve \(3x - 5 < 16\)
\[
\begin{align*}
\quad &+5 &+5 \\
3x &< 21 &
\end{align*}
\]
\[
x < 7
\]

(b) Solve \(\frac{11 - w}{4} = 1 + w\)
\[
\begin{align*}
11 - w &= 4(1 + w) \\
11 - w &= 4 + 4w \\
11 &= 4 + 5w \\
7 &= 5w \\
w &= \frac{7}{5}
\end{align*}
\]
\[
\text{or } 1.4
\]

(Total for Question 17 is 5 marks)
18 (a) Work out \( \frac{1}{5} \times 2\frac{1}{3} \)

Give your answer as a mixed number in its simplest form.

\[
\frac{6}{5} \times \frac{7}{3} = \frac{4 \times 2}{15} = \frac{14}{5} = 2\frac{4}{5}
\]

(b) Work out \( 2\frac{7}{15} - 1\frac{2}{3} \)

\[
\frac{37}{15} - \frac{5 \times 5}{3 \times 5} = \frac{37}{15} - \frac{25}{15} = \frac{12}{15} = \frac{4}{5}
\]

(Total for Question 18 is 6 marks)
19 The diagram shows the graph of \( y = x^3 - 4x - 2 \)

(a) Use the graph to find estimates for the solutions of

(i) \( x^3 - 4x - 2 = 0 \)

(ii) \( x^3 - 4x - 6 = 0 \)

\[ x = -0.4 \quad x = 4.4 \]
\[ x = -1.1 \quad x = 5.1 \]

(b) Use the graph to find estimates for the values of \( x \) that satisfy the simultaneous equations

\[
\begin{align*}
y &= x^3 - 4x - 2 \\
x + y &= 6
\end{align*}
\]

\[
\begin{array}{c|cccc}
& 0 & 1 & 2 \\
- \frac{x}{y} & 6 & 5 & 4 \\
\end{array}
\]

\[ x = -1.7 \quad y = 7.7 \]
\[ x = 4.7 \quad y = 1.4 \]

(Total for Question 19 is 6 marks)
$P$, $M$ and $S$ are points on a circle, centre $O$.
$RST$ is a tangent to the circle.

Angle $PSO = 48^\circ$
$MP = MS$

Work out the size of angle $MST$.
Give reasons for each stage of your working.

- $OPS = 48^\circ$  \textit{Angles at the base of an isosceles triangle are equal \(\bigcirc\)}
- $POS = 84^\circ$  \textit{Angles in a triangle sum to 180}$^\circ$
- $PMS = 42^\circ$  \textit{Angle at circumference is half angle at centre}$
- $MPS \text{ or } MSP = \frac{180 - 42}{2} = \frac{138}{2} = 69^\circ$  \textit{[As \(\bigcirc\)]}$
- $MSO = 21^\circ$  \textit{69$^\circ$ - 48$^\circ$ = 21$^\circ$}$
- $OST = 90^\circ$  \textit{Tangent meets radius at 90}$^\circ$
- $MST = 90 - 21 = 69^\circ$

\textit{(Total for Question 20 is 5 marks)}
21 The probability that it will rain on a day in June is 0.2

When it rains the probability that my tennis match is cancelled is 0.7

When it does not rain, the probability that my tennis match is not cancelled is 0.95

(a) Complete the probability tree diagram for this information.

(b) Work out the probability that, on a day in June, it does not rain and my tennis match is cancelled.

\[
0.8 \times 0.05 = 0.04
\]

(Total for Question 21 is 5 marks)
22. Solve \( x^2 = 4(x - 3)^2 \)

\[
\begin{align*}
  x^2 &= 4 \left( (x - 3)(x - 3) \right) \\
  x^2 &= 4 \left( x^2 - 6x + 9 \right) \\
  x^2 &= 4x^2 - 24x + 36 \\
  0 &= 3x^2 - 24x + 36 \\
  0 &= x^2 - 8x + 12 \\
  0 &= (x - 6)(x - 2) \\
  x &= 6 \quad x = 2
\end{align*}
\]

\( x = 6 \quad x = 2 \)

(Total for Question 22 is 3 marks)
$OPTR$ is a trapezium.

$\overrightarrow{OP} = a$

$\overrightarrow{PT} = b$

$\overrightarrow{OR} = 3b$

(a) (i) Find $\overrightarrow{OT}$ in terms of $a$ and $b$

(b) Find $\overrightarrow{PR}$ in terms of $a$ and $b$

Give your answer in its simplest form.
$S$ is the point on $PR$ such that $PS : SR = 1 : 3$

(b) Find $\overrightarrow{OS}$ in terms of $a$ and $b$
   Give your answer in its simplest form.

\[
\overrightarrow{OS} = \overrightarrow{OP} + \frac{1}{4} \overrightarrow{PR}
\]
\[
= a + \frac{1}{4} (-a + 3b)
\]
\[
= a - \frac{1}{4} a + \frac{3}{4} b
\]
\[
= \frac{3}{4} a + \frac{3}{4} b
\]

\[
\frac{3}{4} a + \frac{3}{4} b
\]

*(c) What does your answer to part (b) tell you about the position of point $S$?

It is \(\frac{3}{4}\) of the way from $O$.

(Total for Question 23 is 6 marks)
24 Given that \( y \propto \frac{1}{x^2} \), complete this table of values.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>100</td>
<td>25</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
y = \frac{k}{x^2}
\]

\[
1 = \frac{k}{(10)^2}
\]

\[
1 = \frac{k}{100}
\]

\[
k = 100
\]

\[
y = \frac{100}{x^2}
\]

\[
\frac{100}{5^2} = 4 \quad \frac{100}{2^2} = 25 \quad \frac{100}{1^2} = 100
\]

(Total for Question 24 is 4 marks)
25 \( ABD \) is a right angled triangle.

\[
\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}
\]

All measurements are given in centimetres.

\( C \) is the point on \( BD \) such that \( CD = \frac{\sqrt{3}}{3} \)

\( AD = BD = \frac{\sqrt{2}}{2} \)

Work out the exact area, in cm\(^2\), of the shaded region.

**Big triangle**

\[
\text{Big triangle} = \frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{8} = \frac{1}{4}
\]

**Little triangle**

\[
\text{Little triangle} = \frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{3} = \frac{\sqrt{6}}{12}
\]

Shaded Area = \( \frac{1}{4} - \frac{\sqrt{6}}{12} \)

\( \frac{1}{4} - \frac{\sqrt{6}}{12} \text{ cm}^2 \)

(Total for Question 25 is 3 marks)