# AS Level Maths: Differentiation

**1** $y = 2x^3 + 5x^2 - 7x + 10$

(a) Find $\dfrac{dy}{dx}$ **(2)**

(b) Find the gradient of the curve when $x = 2$ **(1)**

**(Total for question 1 is 3 marks)**

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**2** $y = 3x + \dfrac{1}{x}$

(a) Find $\dfrac{dy}{dx}$ **(2)**

(b) Find the $x$ coordinates of points where the gradient is zero. **(2)**

**(Total for question 2 is 4 marks)**

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**3** $f(x) = 3x^{\frac{3}{2}} + \dfrac{3}{x^2} - 6x$

Find $f'(x)$

**(Total for question 3 is 4 marks)**

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**4** $y = 4\sqrt{x} + \dfrac{1}{2x} + 10$

(a) Find $\dfrac{dy}{dx}$ **(3)**

(b) Find $\dfrac{d^2y}{dx^2}$ **(2)**

**(Total for question 4 is 5 marks)**

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**5** $y = \dfrac{2x^2 - 5x + 3}{x}$

(a) Find $\dfrac{dy}{dx}$ **(3)**

(b) Find the gradient when $x = 3$ **(1)**

**(Total for question 5 is 4 marks)**

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**6** $y = x^3 - 4x^2 - 3x + 9$

(a) Find $\dfrac{dy}{dx}$ **(2)**

(b) Find the range values of $x$ for which $y$ is increasing **(3)**

**(Total for question 6 is 5 marks)**

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**7** A curve has the equation $y = 2x^3 + 9x^2 - 24x + 13$
Find the coordinates of the curve's local maximum.

**(Total for question 7 is 6 marks)**

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**8**

$$y = 4x^2 + \frac{16}{x} + 1 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ **(3)**

(b) Find the exact range of values of $x$ for which the curve is increasing. **(2)**

**(Total for question 8 is 5 marks)**

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**9** A curve has the equation $y = 2x^3 - 12x^2 + 18x + 5$

(a) The curve has a local minimum at $P$, find the coordinates of $P$. **(4)**

(b) Justify that $P$ is a minimum point. **(2)**

**(Total for question 9 is 6 marks)**

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**10** A curve has the equation $y = 3x^2 - 5x + 7$

Find the equation of the tangent to the curve at the point $P(2, 9)$.

Write your answer in the form $y = mx + c$, where $m$ and $c$ are integers to be found.

**(Total for question 10 is 5 marks)**

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**11** A curve has the equation $y = g(x)$

Given that
- $g(x)$ is a cubic expression in which the coefficient of $x^3$ is equal to the coefficient of $x$
- the curve with equation $y = g(x)$ passes through the origin
- the curve with equation $y = g(x)$ has a stationary point at (2, -10)

(a) Find $g(x)$ **(7)**

(b) prove that the stationary point at (2, -10) is a minimum. **(2)**

**(Total for question 11 is 9 marks)**

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**12** State the interval for which $y = \sin x$ is a decreasing function for $0° \leq x \leq 360°$

**(Total for question 12 is 2 marks)**

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**13** State the interval for which $y = \cos x$ is an increasing function for $0° \leq x \leq 360°$

**(Total for question 13 is 2 marks)**

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**14** $y = 2x^3 + 5x^2 - 7x + 10$

Find the equation of the tangent at the point where $x = 1$
Give your answer in the form $y = mx + c$

**(Total for question 14 is 6 marks)**

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**15** $\mathrm{f}(x) = 2x^3 + x^2 - 18x + 2$

The points $A$ and $B$ lie on the curve $y = \mathrm{f}(x)$. The gradient at both $A$ and $B$ is 2.
Find the coordinates of $A$ and $B$.

**(Total for question 15 is 6 marks)**

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**16** $y = \dfrac{(4x-1)(x+2)}{2x}$

Find the equation of the normal at the point when $x = -2$
Give your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers.

**(Total for question 16 is 6 marks)**

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**17** A simple model for the cost of a car journey £$C$ when a car is driven at a steady speed of $v$ mph is

$$C = \frac{4500}{v} + v + 10$$

(a) Use this model to find the value of $v$ which minimises the cost of the journey. **(5)**

(b) Use $\dfrac{\mathrm{d}^2C}{\mathrm{d}v^2}$ to verify that C is a minimum for this value of $v$ **(2)**

(c) Calculate the minimum cost of the journey **(2)**

**(Total for question 17 is 9 marks)**

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**18** A cylinder has a radius $r$ and a height $h$.
The surface area of the cylinder is 500cm$^2$

(a) Show that the volume ($V$ cm$^3$) of the cylinder is given by $V = 250r - \pi r^3$ **(4)**

Given that $r$ varies

(b) Calculate the maximum value of $V$, to the nearest cm$^3$ **(6)**

(c) Justify that the value of $V$ you found is a maximum. **(2)**

**(Total for question 18 is 12 marks)**

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**19** A curve has the equation $y = 4x^3 + 15x^2 - 18x + 5$

Find the coordinates of the stationary points and determine the nature of each stationary point.

**(Total for question 19 is 6 marks)**

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**20** A curve has equation $y = 3x^2 - 16x\sqrt{x} + 18x - 2$ for $x \geq 0$

(a) Prove that the curve has a maximum point at (1, 3) **(9)**
Fully justify your answer.

(b) Find the coordinates of the other stationary point of the curve and state its nature. **(2)**

**(Total for question 20 is 11 marks)**

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**21** A company is designing a cup. The cup will be in the shape of a cylinder with radius $x$ and height $h$.

The cup does not have a lid and must hold 450 ml of liquid.

(a) Show that the surface area of the cup is given by $\pi x^2 + \dfrac{900}{x}$ **(4)**

(b) Find, to 2 decimal places, the value of $x$ that makes the surface area a minimum. **(4)**

(c) Justify that the value of $x$ you found is a minimum. **(2)**

(d) Give a reason why the company may not choose to make a cup with a radius this size. **(1)**

**(Total for question 21 is 11 marks)**

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**22** Prove that the curve with equation

$$y = 4x^5 + 15x^4 + 20x^3 + 7$$

only has one stationary point, stating its coordinates.

**(Total for question 22 is 6 marks)**

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**23** A curve has equation

$$y = 2x^3 - 3x^2 + 4x - 5$$

(a) Find $\dfrac{dy}{dx}$ **(4)**

(b) Show that the perpendicular bisector of the line joining A(6, 2) and B(4, -6) is a normal to the curve at (1, -1). **(6)**

**(Total for question 23 is 10 marks)**

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**24** A curve has equation

$$y = x^3 + px^2 + qx - 5$$

The curve passes through the point $A$ (2, 1)

The gradient of the curve at $A$ is 5.

Find the value of $p$ and the value of $q$.

**(Total for question 24 is 5 marks)**

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**25** A curve has equation $f(x) = (x + 3)(x - 2)^2$

(a) Find the coordinates of the turning points of the curve.
Determine the nature of each turning point. **(8)**

(b) State the coordinates of the turning points of the curve $y = 2f(x - 1)$ **(2)**

**(Total for question 25 is 10 marks)**

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**26** A curve has equation $y = 3x^4 - 2\sqrt{x} + \frac{x}{2} - 2$

Find an expression for $\frac{d^2y}{dx^2}$

**(Total for question 26 is 3 marks)**

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**27** A curve has equation $y = ax^2 - \frac{b}{\sqrt{x}} + \frac{c}{x}$

(a) In terms of a, b and c, find an expression for $\frac{dy}{dx}$ **(4)**

(b) In terms of a, b and c, find an expression for $\frac{d^2y}{dx^2}$ **(3)**

**(Total for question 27 is 7 marks)**

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**28** A curve has equation $y = x^2 - 4x$

(a) Find $\frac{dy}{dx}$ **(2)**

(b) Find the values of $x$ for which $y$ is increasing. **(2)**

**(Total for question 28 is 10 marks)**

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**29** The line $y = 3x + k$ is a tangent to the curve $x^2 - y = 3$. Find the value of the constant $k$.

**(Total for question 29 is 5 marks)**

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**30** Find the equation of the normal to the curve $y = 2\sqrt{x} + 3x + 1$ at the point where $x = 4$.

Give your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(Total for question 30 is 7 marks)**

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**31** Find the equation of the normal to the curve $y = (2x - 1)^2$ at the point where $x = 2$.

Give your answer in the form $y = mx + c$

**(Total for question 31 is 6 marks)**

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**32** (a) Sketch the gradient function of the curve $y = x^3 - 3x^2 - 45x$ **(5)**

(b) Determine the set of values for which $x^3 - 3x^2 - 45x$ is decreasing **(2)**

**(Total for question 32 is 7 marks)**

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**33** The equation of a curve is $y = 2x^2 + \frac{1}{x}$

A tangent and a normal to the curve are drawn at the point where $x = 1$.

Calculate the area bounded by the tangent, the normal and the $x$-axis.

**(Total for question 33 is 9 marks)**

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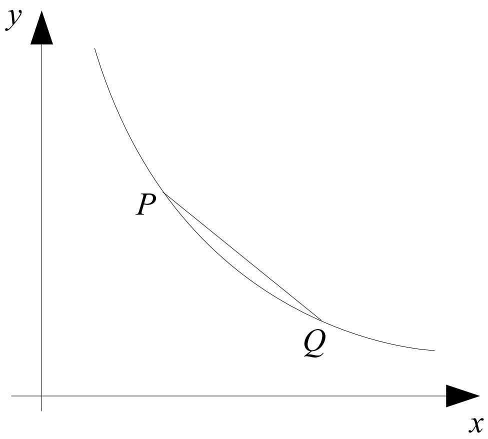
**34** The graph shows part of the curve with equation $y = \frac{4}{x}$




$P$ is the point with coordinates (1, 4) and $Q$ is the point with x coordinate $(1+ h)$

The table shows for different values of $h$, the coordinates of $P$, the coordinates of $Q$ and the gradient of the chord $PQ$

| $x$ for $P$ | $y$ for $P$ | $h$ | $x$ for $Q$ | $y$ for $Q$ | Gradient |
|---|---|---|---|---|---|
| 1 | 4 | 1 | 2 | 2 | -2 |
| 1 | 4 | 0.1 | 1.1 | 3.636364 | -3.636364 |
| 1 | 4 | 0.01 | | | |
| 1 | 4 | 0.001 | | | |

(a) Complete the table. **(3)**

(b) Explain how the sequence of values in the last column relates to the gradient of the curve at the point $P$. **(1)**

(c) Use calculus to find the gradient of the curve at the point P. **(2)**

**(Total for question 34 is 6 marks)**

**35** Danny is investigating the gradient of chords of the curve with equation $f(x) = 2 - x^2$

Each chord joins the point (3, -7) to the point $(3 + h, f(3 + h))$

The table shows Danny's results.

| $x$ | f($x$) | $h$ | $x + h$ | f($x + h$) | Gradient |
|---|---|---|---|---|---|
| 3 | -7 | 1 | 4 | -14 | -7 |
| 3 | -7 | 0.1 | 3.1 | -7.61 | |
| 3 | -7 | 0.01 | | | |
| 3 | -7 | 0.001 | | | |

(a) Complete the table. **(5)**

(b) Suggest the limit for the gradient of these chords as $h$ tends to 0. **(1)**

**(Total for question 35 is 6 marks)**

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**36** A cuboid *ABCDEF* has width $2x$, height $x$ and depth $y$.

The volume of the cuboid is 600 cm$^3$. The surface area of the cuboid is $S$ cm$^2$.

(a) Show that $S = 4x^2 + \dfrac{1800}{x}$ **(5)**

(b) Determine the value of x for which the surface area of the cuboid is a minimum. **(4)**

(c) Find, to the nearest integer, the minimum value of S. **(1)**

**(Total for question 36 is 10 marks)**

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**37** (i) A curve has equation $y = 8x + \dfrac{1}{2x^2}$

(a) Find an expression for $\dfrac{dy}{dx}$ **(2)**

(b) Find an expression for $\dfrac{d^2y}{dx^2}$ **(2)**

(ii) Hence find the coordinates of the stationary point and determine its nature. **(5)**

**(Total for question 37 is 9 marks)**

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**38** Show that the only stationary point on the graph of $y = 2x^2 - 8\sqrt{x}$ is a minimum point at (1, -6)

**(Total for question 38 is 7 marks)**

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**39** Prove, from first principles, that the derivative of $4x$ is 4.

**(Total for question 39 is 4 marks)**

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**40** Prove, from first principles, that the derivative of $x^3$ is $3x^2$.

**(Total for question 40 is 5 marks)**

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**41** Prove, from first principles, that the derivative of $2x^3$ is $6x^2$.

**(Total for question 41 is 5 marks)**

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**42** Prove, from first principles, that the derivative of $5x^2$ is $10x$.

**(Total for question 42 is 4 marks)**

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**43** Prove, from first principles, that the derivative of $kx^3$ is $3kx^2$.
Where $k$ is a constant.

**(Total for question 43 is 5 marks)**

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**44** A curve $C$ has equation $y = 3x^2 + 1$

The point $P(3, 28)$ lies on the curve.

(a) Find the gradient of the tangent at $P$. **(2)**

The point $Q$ with $x$-coordinate $(3 + h)$ also lies on C.

(b) Find the gradient of the line $PQ$, giving your answer in terms of $h$ in its simplest form. **(3)**

(c) Explain briefly the relationship between part (b) and the answer to part (a). **(1)**

**(Total for question 44 is 6 marks)**

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**45** Differentiate $3x^2 + x$ from first principles.

**(Total for question 45 is 5 marks)**

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**46** Differentiate $4x - 3x^2$ from first principles.

**(Total for question 46 is 5 marks)**

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**47** (a) Sketch the gradient function of the curve $y = x^3 + 3x^2 - 24x$ **(5)**

(b) Determine the set of values for which $x^3 + 3x^2 - 24x$ is increasing **(2)**

**(Total for question 47 is 7 marks)**

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**48** A curve has equation $y = x^3 + x^2$

A normal to the curve is drawn at the point where $x = 1$ and meets the $x$-axis at $A$ and the $y$-axis at $B$.

(a) Find the area of $OAB$. **(6)**

(b) Use calculus to prove the curve has one local maximum and one local minimum point. **(6)**

**(Total for question 48 is 12 marks)**

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**49** The equation of a curve is $y = 4\sqrt{x} - 8x^2$

(a) Find $\frac{dy}{dx}$ **(3)**

(b) Find the coordinates of the turning point. **(3)**

(c) Determine the nature of the turning point. **(2)**

**(Total for question 49 is 8 marks)**

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