1

a $\frac{dy}{dx} = 2x + 6$
$2x + 6 = 0$
$x = -3$

b $\frac{dy}{dx} = 8x + 2$
$8x + 2 = 0$
$x = -\frac{1}{4}$

c $\frac{dy}{dx} = 3x^2 - 12$
$3x^2 - 12 = 0$
$x^2 = 4$
$x = \pm 2$

d $\frac{dy}{dx} = 18x - 3x^2$
$18x - 3x^2 = 0$
$3x(6 - x) = 0$
$x = 0, 6$

e $\frac{dy}{dx} = 3x^2 - 10x + 3$
$3x^2 - 10x + 3 = 0$
$(3x - 1)(x - 3) = 0$
$x = \frac{1}{3}, 3$

f $\frac{dy}{dx} = 1 - 9x^{-2}$
$1 - 9x^{-2} = 0$
$x^2 = 9$
$x = \pm 3$

g $y = x^3 - 3x^2 + 3x - 9$
$\frac{dy}{dx} = 3x^2 - 6x + 3$
$3x^2 - 6x + 3 = 0$
$3(x - 1)^2 = 0$
$x = 1$

h $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 2$
$\frac{1}{2}x^{-\frac{1}{2}} - 2 = 0$
$x^{-\frac{1}{2}} = 4$
$x = \frac{1}{16}$

2

a $f'(x) = 4x + 2$
$\therefore\ 4x + 2 \geq 0$
$x \geq -\frac{1}{2}$

b $f'(x) = 6x - 6x^2$
$\therefore\ 6x - 6x^2 \geq 0$
$6x(1 - x) \geq 0$
$0 \leq x \leq 1$

c $f'(x) = 9x^2 - 1$
$\therefore\ 9x^2 - 1 \geq 0$
$x^2 \geq \frac{1}{9}$
$x \leq -\frac{1}{3}$ and $x \geq \frac{1}{3}$

d $f'(x) = 3x^2 + 12x - 15$
$\therefore\ 3x^2 + 12x - 15 \geq 0$
$3(x + 5)(x - 1) \geq 0$
$x \leq -5$ and $x \geq 1$

e $f(x) = x^3 - 12x^2 + 36x$
$f'(x) = 3x^2 - 24x + 36$
$\therefore\ 3x^2 - 24x + 36 \geq 0$
$3(x - 2)(x - 6) \geq 0$
$x \leq 2$ and $x \geq 6$

f $f'(x) = 2 - 8x^{-2}$
$\therefore\ 2 - 8x^{-2} \geq 0$
$x^2 \geq 4$
$x \leq -2$ and $x \geq 2$

3

a $f'(x) = 3x^2 + 4x$
$\therefore\ 3x^2 + 4x \leq 0$
$x(3x + 4) \leq 0$
$-\frac{4}{3} \leq x \leq 0$

b $f'(x) = 27 - 3x^2$
$\therefore\ 27 - 3x^2 \leq 0$
$x^2 \geq 9$
$x \leq -3$ and $x \geq 3$

c $f(x) = 2x^3 - x^2 - 4x + 2$
$f'(x) = 6x^2 - 2x - 4$
$\therefore\ 6x^2 - 2x - 4 \leq 0$
$2(3x + 2)(x - 1) \leq 0$
$-\frac{2}{3} \leq x \leq 1$

4

a $(x + 1)$ factor $\therefore$ $f(-1) = 0$
$\therefore\ -1 + k + 3 = 0$
$k = -2$

b $f'(x) = 3x^2 - 4x$
$\therefore\ 3x^2 - 4x \geq 0$
$x(3x - 4) \geq 0$
$x \leq 0$ and $x \geq \frac{4}{3}$

5 **a** $\frac{dy}{dx} = 2x + 2$

SP: $2x + 2 = 0$

$x = -1$

$\therefore \ (-1, -1)$

b $\frac{dy}{dx} = 10x - 4$

SP: $10x - 4 = 0$

$x = \frac{2}{5}$

$\therefore \ (\frac{2}{5}, \frac{1}{5})$

c $\frac{dy}{dx} = 3x^2 - 3$

SP: $3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$

$\therefore \ (-1, 6), (1, 2)$

d $\frac{dy}{dx} = 12x^2 + 6x$

SP: $12x^2 + 6x = 0$

$6x(2x + 1) = 0$

$x = -\frac{1}{2}, 0$

$\therefore \ (-\frac{1}{2}, \frac{9}{4}), (0, 2)$

e $\frac{dy}{dx} = 2 - 8x^{-2}$

SP: $2 - 8x^{-2} = 0$

$x^2 = 4$

$x = \pm 2$

$\therefore \ (-2, -5), (2, 11)$

f $\frac{dy}{dx} = 3x^2 - 18x - 21$

SP: $3x^2 - 18x - 21 = 0$

$3(x + 1)(x - 7) = 0$

$x = -1, 7$

$\therefore \ (-1, 22), (7, -234)$

g $\frac{dy}{dx} = -x^{-2} - 8x$

SP: $-x^{-2} - 8x = 0$

$x^3 = -\frac{1}{8}$

$x = -\frac{1}{2}$

$\therefore \ (-\frac{1}{2}, -3)$

h $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 6$

SP: $3x^{\frac{1}{2}} - 6 = 0$

$x^{\frac{1}{2}} = 2$

$x = 4$

$\therefore \ (4, -8)$

i $\frac{dy}{dx} = 6x^{-\frac{1}{3}} - 2$

SP: $6x^{-\frac{1}{3}} - 2 = 0$

$x^{-\frac{1}{3}} = 3$

$x = \frac{1}{27}$

$\therefore \ (\frac{1}{27}, 5\frac{25}{27})$

6 **a** $\frac{dy}{dx} = 4 - 2x$

SP: $4 - 2x = 0$

$x = 2$

$\frac{d^2y}{dx^2} = -2$

(2, 9): max

b $\frac{dy}{dx} = 3x^2 - 3$

SP: $3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$

$\frac{d^2y}{dx^2} = 6x$

$(-1, 2)$: $\frac{d^2y}{dx^2} = -6$, max

$(1, -2)$: $\frac{d^2y}{dx^2} = 6$, min

c $\frac{dy}{dx} = 3x^2 + 18x$

SP: $3x^2 + 18x = 0$

$3x(x + 6) = 0$

$x = -6, 0$

$\frac{d^2y}{dx^2} = 6x + 18$

$(-6, 100)$: $\frac{d^2y}{dx^2} = -18$, max

$(0, -8)$: $\frac{d^2y}{dx^2} = 18$, min

d $\frac{dy}{dx} = 3x^2 - 12x - 36$

SP: $3x^2 - 12x - 36 = 0$

$3(x + 2)(x - 6) = 0$

$x = -2, 6$

$\frac{d^2y}{dx^2} = 6x - 12$

$(-2, 55)$: $\frac{d^2y}{dx^2} = -24$, max

$(6, -201)$: $\frac{d^2y}{dx^2} = 24$, min

e $\frac{dy}{dx} = 4x^3 - 16x$

SP: $4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$x = 0, \pm 2$

$\frac{d^2y}{dx^2} = 12x^2 - 16$

$(-2, -18)$: $\frac{d^2y}{dx^2} = 32$, min

$(0, -2)$: $\frac{d^2y}{dx^2} = -16$, max

$(2, -18)$: $\frac{d^2y}{dx^2} = 32$, min

f $\frac{dy}{dx} = 9 - 4x^{-2}$

SP: $9 - 4x^{-2} = 0$

$x^2 = \frac{4}{9}$

$x = \pm \frac{2}{3}$

$\frac{d^2y}{dx^2} = 8x^{-3}$

$(-\frac{2}{3}, -12)$: $\frac{d^2y}{dx^2} = -27$, max

$(\frac{2}{3}, 12)$: $\frac{d^2y}{dx^2} = 27$, min

g $\frac{dy}{dx} = 1 - 3x^{-\frac{1}{2}}$

SP: $1 - 3x^{-\frac{1}{2}} = 0$

$x^{-\frac{1}{2}} = \frac{1}{3}$

$x = 9$

$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{3}{2}}$

$(9, -9)$: $\frac{d^2y}{dx^2} = \frac{1}{18}$, min

h $\frac{dy}{dx} = -8 + 14x - 6x^2$

SP: $-8 + 14x - 6x^2 = 0$

$-2(3x - 4)(x - 1) = 0$

$x = 1, \frac{4}{3}$

$\frac{d^2y}{dx^2} = 14 - 12x$

$(1, 0)$: $\frac{d^2y}{dx^2} = 2$, min

$(\frac{4}{3}, \frac{1}{27})$: $\frac{d^2y}{dx^2} = -2$, max

i $y = \frac{1}{2}x^2 + 8x^{-2}$

$\frac{dy}{dx} = x - 16x^{-3}$

SP: $x - 16x^{-3} = 0$

$x^4 = 16$

$x = \pm 2$

$\frac{d^2y}{dx^2} = 1 + 48x^{-4}$

$(-2, 4)$: $\frac{d^2y}{dx^2} = 4$, min

$(2, 4)$: $\frac{d^2y}{dx^2} = 4$, min

7 **a** $\frac{dy}{dx} = 2x - 3x^2$

SP: $2x - 3x^2 = 0$

$x(2 - 3x) = 0$

$x = 0, \frac{2}{3}$

$\frac{d^2y}{dx^2} = 2 - 6x$

$(0, 0)$: $\frac{d^2y}{dx^2} = 2$, min

$(\frac{2}{3}, \frac{4}{27})$: $\frac{d^2y}{dx^2} = -2$, max

b $\frac{dy}{dx} = 3x^2 + 6x + 3$

SP: $3x^2 + 6x + 3 = 0$

$3(x + 1)^2 = 0$

$x = -1$

$\frac{d^2y}{dx^2} = 6x + 6$

$(-1, -1)$: $\frac{d^2y}{dx^2} = 0$

x	< -1	-1	> -1
$\frac{dy}{dx}$	+	0	+

$\therefore$ $(-1, -1)$: point of inflexion

c $\frac{dy}{dx} = 4x^3$

SP: $4x^3 = 0$

$x = 0$

$\frac{d^2y}{dx^2} = 12x^2$

$(0, -2)$: $\frac{d^2y}{dx^2} = 0$

x	< 0	0	> 0
$\frac{dy}{dx}$	–	0	+

$\therefore$ $(0, -2)$: min

d $\frac{dy}{dx} = -12 + 12x - 3x^2$

SP: $-12 + 12x - 3x^2 = 0$

$-3(x - 2)^2 = 0$

$x = 2$

$\frac{d^2y}{dx^2} = 12 - 6x$

$(2, -4)$: $\frac{d^2y}{dx^2} = 0$

x	< 2	2	> 2
$\frac{dy}{dx}$	–	0	–

$\therefore$ $(2, -4)$: point of inflexion

e $\frac{dy}{dx} = 2x - 16x^{-2}$

SP: $2x - 16x^{-2} = 0$

$x^3 = 8$

$x = 2$

$\frac{d^2y}{dx^2} = 2 + 32x^{-3}$

$(2, 12)$: $\frac{d^2y}{dx^2} = 6$, min

f $\frac{dy}{dx} = 4x^3 + 12x^2$

SP: $4x^3 + 12x^2 = 0$

$4x^2(x + 3) = 0$

$x = -3, 0$

$\frac{d^2y}{dx^2} = 12x^2 + 24x$

$(-3, -28)$: $\frac{d^2y}{dx^2} = 36$, min

$(0, -1)$: $\frac{d^2y}{dx^2} = 0$

x	$-3 < x < 0$	0	> 0
$\frac{dy}{dx}$	+	0	+

$\therefore$ $(0, -1)$: point of inflexion

8 **a** $\frac{dy}{dx} = 3x^2 + 6x$

SP: $3x^2 + 6x = 0$

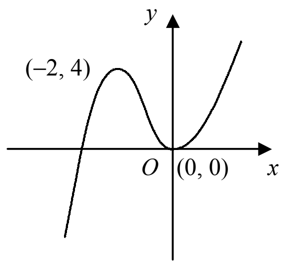
$3x(x + 2) = 0$

$x = -2, 0$

$\frac{d^2y}{dx^2} = 6x + 6$

$(-2, 4)$: $\frac{d^2y}{dx^2} = -6$, max

$(0, 0)$: $\frac{d^2y}{dx^2} = 6$, min



b $\frac{dy}{dx} = 1 - x^{-2}$

SP: $1 - x^{-2} = 0$

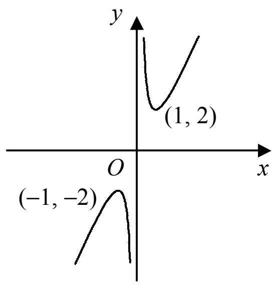
$x^2 = 1$

$x = \pm 1$

$\frac{d^2y}{dx^2} = 2x^{-3}$

$(-1, -2)$: $\frac{d^2y}{dx^2} = -2$, max

$(1, 2)$: $\frac{d^2y}{dx^2} = 2$, min



c $\frac{dy}{dx} = 3x^2 - 6x + 3$

SP: $3x^2 - 6x + 3 = 0$

$3(x - 1)^2 = 0$

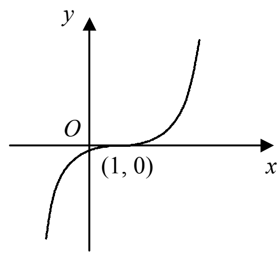
$x = 1$

$\frac{d^2y}{dx^2} = 6x - 6$

$(1, 0)$: $\frac{d^2y}{dx^2} = 0$

x	< 1	1	> 1
$\frac{dy}{dx}$	$+$	0	$+$

$\therefore$ $(1, 0)$: point of inflexion



d $\frac{dy}{dx} = 3 - 2x^{-\frac{1}{2}}$

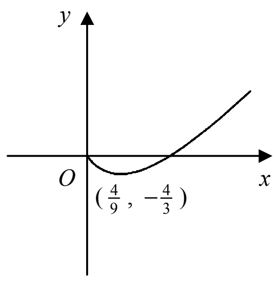
SP: $3 - 2x^{-\frac{1}{2}} = 0$

$x^{-\frac{1}{2}} = \frac{3}{2}$

$x = \frac{4}{9}$

$\frac{d^2y}{dx^2} = x^{-\frac{3}{2}}$

$(\frac{4}{9}, -\frac{4}{3})$: $\frac{d^2y}{dx^2} = \frac{27}{8}$, min



e $\frac{dy}{dx} = 3x^2 + 8x - 3$

SP: $3x^2 + 8x - 3 = 0$

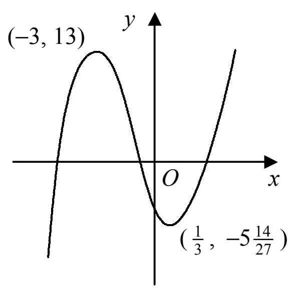
$(3x - 1)(x + 3) = 0$

$x = -3, \frac{1}{3}$

$\frac{d^2y}{dx^2} = 6x + 8$

$(-3, 13)$: $\frac{d^2y}{dx^2} = -10$, max

$(\frac{1}{3}, -5\frac{14}{27})$: $\frac{d^2y}{dx^2} = 10$, min



f $y = x^4 - 8x^2 + 12$

$\frac{dy}{dx} = 4x^3 - 16x$

SP: $4x^3 - 16x = 0$

$4x(x + 2)(x - 2) = 0$

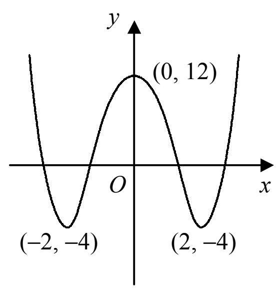
$x = -2, 0, 2$

$\frac{d^2y}{dx^2} = 12x^2 - 16$

$(-2, -4)$: $\frac{d^2y}{dx^2} = 32$, min

$(0, 12)$: $\frac{d^2y}{dx^2} = -16$, max

$(2, -4)$: $\frac{d^2y}{dx^2} = 32$, min



1 **a** volume $= 2x^2h = 4000$

$\therefore\ h = \frac{2000}{x^2}$

b $A = 2x^2 + 2(2xh) + 2(xh)$

$= 2x^2 + 6xh$

$= 2x^2 + (6x \times \frac{2000}{x^2})$

$= 2x^2 + \frac{12\,000}{x}$

c $\frac{dA}{dx} = 4x - 12\,000x^{-2}$

SP: $4x - 12\,000x^{-2} = 0$

$x^3 = 3000$

$x = \sqrt[3]{3000} = 14.4$ (3sf)

d min $A = 1250$ (3sf)

e $\frac{d^2A}{dx^2} = 4 + 24\,000x^{-3}$

when $x = \sqrt[3]{3000}$, $\frac{d^2A}{dx^2} = 12$

$\frac{d^2A}{dx^2} > 0\ \therefore$ minimum

2 **a** S.A. $= 2\pi r^2 + 2\pi rh = 30\,000$

$\therefore\ \pi rh = 15\,000 - \pi r^2$

$h = \frac{15\,000}{\pi r} - r$

$V = \pi r^2 h$

$= \pi r^2(\frac{15\,000}{\pi r} - r)$

$= 15\,000r - \pi r^3$

b $\frac{dV}{dr} = 15\,000 - 3\pi r^2$

SP: $15\,000 - 3\pi r^2 = 0$

$r^2 = \frac{5000}{\pi}$

$r = \sqrt{\frac{5000}{\pi}}$ [= 39.9 (3sf)]

max volume = 399 000 cm^3 (3sf)

$\frac{d^2V}{dr^2} = -6\pi r$

when $r = \sqrt{\frac{5000}{\pi}}$, $\frac{d^2V}{dr^2} = -752$

$\frac{d^2V}{dr^2} < 0\ \therefore$ maximum

3 **a** S.A. $= 2x^2 + 4xl = k$

$\therefore\ 4xl = k - 2x^2$

$l = \frac{k - 2x^2}{4x}$

b $V = x^2 l$

$= x^2 \times \frac{k - 2x^2}{4x}$

$= \frac{1}{4}kx - \frac{1}{2}x^3$

$\frac{dV}{dx} = \frac{1}{4}k - \frac{3}{2}x^2$

SP: $\frac{1}{4}k - \frac{3}{2}x^2 = 0$

$x^2 = \frac{1}{6}k$

$x = \sqrt{\frac{k}{6}}$

$\frac{d^2V}{dx^2} = -3x$

when $x = \sqrt{\frac{k}{6}}$, $\frac{d^2V}{dx^2} < 0\ \therefore$ maximum

$l = \frac{k - \frac{1}{3}k}{4\sqrt{\frac{k}{6}}} = \frac{2}{3}k \times \frac{1}{4} \times \sqrt{\frac{6}{k}}$

$= \frac{k}{6} \times \sqrt{\frac{6}{k}} = \sqrt{\frac{k}{6}}$

$\therefore$ maximum V when $l = x\ \therefore$ prism is a cube

1 **a** $f'(x) = 6x^2 + 10x$

b $6x^2 + 10x \geq 0$
$2x(3x + 5) \geq 0$
$x \leq -\frac{5}{3}$ and $x \geq 0$

2 **a** $\frac{dy}{dx} = 3x^2 - 2x + 2$
at (1, −2), grad = 3
$\therefore\ y + 2 = 3(x - 1)$
$3x - y - 5 = 0$

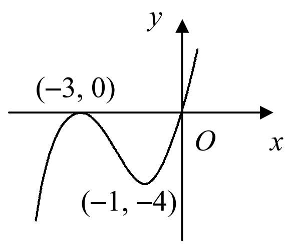
b SP when $3x^2 - 2x + 2 = 0$
$b^2 - 4ac = 4 - 24 = -20$
$b^2 - 4ac < 0$ ∴ no real roots
∴ no stationary points

3 **a** $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 4x^{-2}$
$\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-\frac{3}{2}} + 8x^{-3}$

b SP: $\frac{1}{2}x^{-\frac{1}{2}} - 4x^{-2} = 0$
$\frac{1}{2}x^{-2}(x^{\frac{3}{2}} - 8) = 0$
$x^{\frac{3}{2}} = 8$
$x = 4$
∴ (4, 3)
when $x = 4$, $\frac{d^2y}{dx^2} = \frac{3}{32}$
$\frac{d^2y}{dx^2} > 0$ ∴ minimum

4 **a** $y = 0 \Rightarrow x(x + 3)^2 = 0$
$x = -3, 0$
∴ (−3, 0), (0, 0)

b $f'(x) = 3x^2 + 12x + 9$
decreasing when $3x^2 + 12x + 9 \leq 0$
$3(x + 3)(x + 1) \leq 0$
$\therefore\ -3 \leq x \leq -1$

c


5 **a** $\frac{dh}{dt} = 8t^3 - 24t^2 + 16t$

b when $t = 0.25$,
$\frac{dh}{dt} = 2.625$ cm per second

c SP: $8t^3 - 24t^2 + 16t = 0$
$8t(t - 1)(t - 2) = 0$
$t = 0, 1, 2$
from graph, max when $t = 1$
∴ max height = 3 cm

6 **a** $\frac{dy}{dx} = 3x^2 + 6kx - 9k^2$
stationary when $3x^2 + 6kx - 9k^2 = 0$
$\Rightarrow x^2 + 2kx - 3k^2 = 0$

b $(x + 3k)(x - k) = 0$
$x = -3k, k$
when $x = k$, $y = k^3 + 3k^3 - 9k^3 = -5k^3$
∴ stationary at $(k, -5k^3)$

c when $x = -3k$,
$y = -27k^3 + 27k^3 + 27k^3 = 27k^3$
∴ $(-3k, 27k^3)$

7 **a** $V = \frac{1}{2}x^2 \sin 60° \times l$

$= \frac{1}{2}x^2 l \times \frac{\sqrt{3}}{2} = 250$

$\therefore\ l = \frac{1000}{\sqrt{3}x^2}$ or $\frac{1000\sqrt{3}}{3x^2}$

b $A = (2 \times \frac{\sqrt{3}}{4}x^2) + 3xl$

$= \frac{\sqrt{3}}{2}x^2 + (3x \times \frac{1000\sqrt{3}}{3x^2})$

$= \frac{\sqrt{3}}{2}(x^2 + \frac{2000}{x})$

c $\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{\sqrt{3}}{2}(2x - 2000x^{-2})$

SP: $\frac{\sqrt{3}}{2}(2x - 2000x^{-2}) = 0$

$x^3 = 1000$

$x = 10$

d min $A = 150\sqrt{3}$

e $\frac{\mathrm{d}^2A}{\mathrm{d}x^2} = \frac{\sqrt{3}}{2}(2 + 4000x^{-3})$

when $x = 10$, $\frac{\mathrm{d}^2A}{\mathrm{d}x^2} = 3\sqrt{3}$

$\frac{\mathrm{d}^2A}{\mathrm{d}x^2} > 0\ \therefore$ minimum

8 **a** $f'(x) = 3x^2 + 8x + k$

for 2 SPs, $f'(x) = 0$ has 2 distinct roots

$\therefore\ b^2 - 4ac > 0$

$64 - 12k > 0$

$k < \frac{16}{3}$

b SP: $3x^2 + 8x - 3 = 0$

$(3x - 1)(x + 3) = 0$

$x = -3, \frac{1}{3}$

$\therefore\ (-3, 19)$ and $(\frac{1}{3}, \frac{13}{27})$

9 **a** $x^{\frac{1}{2}} - 4 + 3x^{-\frac{1}{2}} = 0$

$x - 4x^{\frac{1}{2}} + 3 = 0$

$(x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 3) = 0$

$x^{\frac{1}{2}} = 1, 3$

$x = 1, 9$

$\therefore\ (1, 0)$ and $(9, 0)$

b $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$

SP: $\frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} = 0$

$\frac{1}{2}x^{-\frac{3}{2}}(x - 3) = 0$

$x = 3$

$y = \sqrt{3} - 4 + \frac{3}{\sqrt{3}} = 2\sqrt{3} - 4$

$\therefore\ (3, 2\sqrt{3} - 4)$

10 **a** $f(-1) = -1 - 3 + 4 = 0$

$\therefore\ (x + 1)$ is a factor

b

$$\begin{array}{r|l} & x^2 - 4x + 4 \\ \hline x+1 & x^3 - 3x^2 + 0x + 4 \\ & \underline{x^3 + x^2} \\ & -4x^2 + 0x \\ & \underline{-4x^2 - 4x} \\ & 4x + 4 \\ & \underline{4x + 4} \end{array}$$

$\therefore\ f(x) \equiv (x + 1)(x^2 - 4x + 4)$

$f(x) \equiv (x + 1)(x - 2)^2$

c $(2, 0)$, as $(x - 2)$ is a repeated factor of $f(x)$ so x-axis is a tangent at $(2, 0)$

d $f'(x) = 3x^2 - 6x$

SP: $3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0, 2$

$\therefore\ (0, 4)$ is other turning point