

# Partial Fractions

## Splitting Fractions

$$\frac{6x-2}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$\frac{6x-2}{(x-3)(x+1)} = \frac{A(x+1)}{(x-3)(x+1)} + \frac{B(x-3)}{(x-3)(x+1)}$$

let  $x = -1$       $6(-1) - 2 = A((-1) + 1) + B((-1) - 3)$

$$B = 2$$

let  $x = 3$       $6(3) - 2 = A((3) + 1) + B((3) - 3)$

$$A = 4$$

When there is a square in the denominator, it is included twice:

$$\frac{6x-2}{(x-3)(x+1)^2} = \frac{A}{x-3} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

## Top Heavy Fractions

For top heavy fractions divide the numerator by the denominator algebraic long division).

# The Binomial Expansion

We can expand negative and fractional powers using the formula:

$$(1+kx)^n = 1 + n(kx) + \frac{n(n-1)(kx)^2}{2 \times 1} + \frac{n(n-1)(n-2)(kx)^3}{3 \times 2 \times 1}$$

If the number at the front is not 1,  
factorise out first

$$(4-3x)^{-\frac{1}{2}}$$
$$4^{-\frac{1}{2}} \left(1 + \frac{3}{4}x\right)^{-\frac{1}{2}}$$

$$\frac{1}{2} \left(1 + \frac{3}{4}x\right)^{-\frac{1}{2}}$$

$$\frac{1}{2} \left(1 + \left(\frac{-1}{2}\right) \left(\frac{3}{4}x\right) + \frac{\left(\frac{-1}{2}\right) \left(\frac{-3}{2}\right) \left(\frac{3}{4}x\right)^2}{2 \times 1} + \frac{\left(\frac{-1}{2}\right) \left(\frac{-3}{2}\right) \left(\frac{-5}{2}\right) \left(\frac{3}{4}x\right)^3}{3 \times 2 \times 1}\right)$$

$$\frac{1}{2} \left(1 - \frac{3}{8}x + \frac{27}{128}x^2 - \frac{135}{1024}x^3\right)$$

$$\frac{1}{2} - \frac{3}{16}x + \frac{27}{256}x^2 - \frac{135}{2048}x^3$$