Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Differentiation and Integration

Materials required for examination
Mathematical Formulae (Pink or Green)

Items included with question papers
Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A river, running between parallel banks, is 20 m wide. The depth, y metres, of the river measured at a point x metres from one bank is given by the formula

$$y = \frac{1}{10}x\sqrt{(20-x)}, \qquad 0 \le x \le 20.$$

(*a*) Complete the table below, giving values of y to 3 decimal places.

x	0	4	8	12	16	20
y	0	1.6	2.771	3.394	3.2	0

(2)

(*b*) Use the trapezium rule with all the values in the table to estimate the cross-sectional area of the river.

(4)

Given that the cross-sectional area is constant and that the river is flowing uniformly at 2 m s^{-1},

(*c*) estimate, in m^3, the volume of water flowing per minute, giving your answer to 3 significant figures.

(2)

b/ $4(1.6 + 2.771 + 3.394 + 3.2)$

$= \underline{\underline{43.86 \text{ m}^2}}$

c) $43.86 \times 120 = \cancel{87.7 \text{ m}^3}$

2 m per second
120 m per minute

$= 5263.2 \text{ m}^3$

$= \underline{\underline{5260 \text{ m}^3}}$ (3sf)

2. The speed, v m s^{-1}, of a train at time t seconds is given by

$$v = \sqrt{(1.2^t - 1)}, \qquad 0 \leq t \leq 30.$$

The following table shows the speed of the train at 5 second intervals.

t	0	5	10	15	20	25	30
v	0	1.22	2.28	3.80	6.11	9.72	15.37

(a) Complete the table, giving the values of v to 2 decimal places.

(3)

The distance, s metres, travelled by the train in 30 seconds is given by

$$s = \int_0^{30} \sqrt{(1.2^t - 1)}\ \mathrm{d}t.$$

(b) Use the trapezium rule, with all the values from your table, to estimate the value of s.

(3)

$$5\left(\frac{0}{2} + 1.22 + 2.28 + 3.80 + 6.11 + 9.72 + \frac{15.37}{2}\right)$$

$$= \underline{\underline{154.075}} \text{ m}$$

3. (*a*) Sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the y-axis.

(2)

(*b*) Copy and complete the table, giving the values of 3^x to 3 decimal places.

x	0	0.2	0.4	0.6	0.8	1
3^x	1	1.246	1.552	1.933	2.408	3

(2)

(*c*) Use the trapezium rule, with all the values from your tables, to find an approximation for the value of $\int_0^1 3^x \, dx$.

(4)

$0.2\left(\frac{1}{2} + 1.246 + 1.552 + 1.933 + 2.408 + \frac{3}{2}\right)$

$= 1.8278$ units^2

4. The curve C has equation

$$y = x\sqrt{(x^3 + 1)}, \qquad 0 \leq x \leq 2.$$

(a) Copy and complete the table below, giving the values of y to 3 decimal places at $x = 1$ and $x = 1.5$.

x	0	0.5	1	1.5	2
y	0	0.530	1.414	3.137	6

(2)

(b) Use the trapezium rule, with all the y values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3 + 1)}\, dx$, giving your answer to 3 significant figures.

(4)

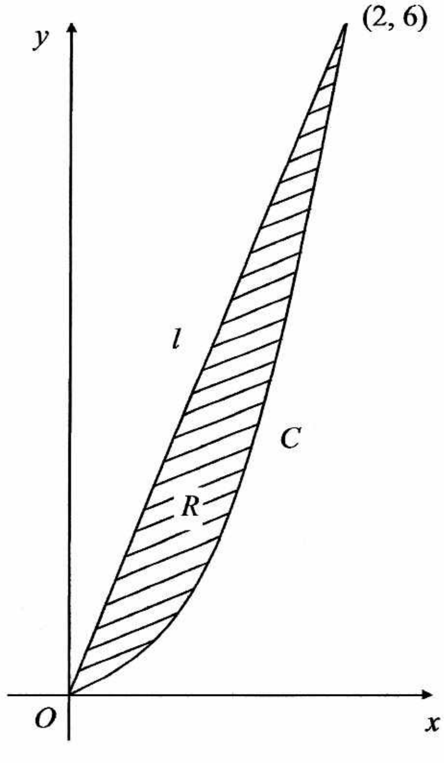

Figure 2

Figure 2 shows the curve C with equation $y = x\sqrt{(x^3 + 1)}$, $0 \leq x \leq 2$, and the straight line segment l, which joins the origin and the point (2, 6). The finite region R is bounded by C and l.

(c) Use your answer to part (b) to find an approximation for the area of R, giving your answer to 3 significant figures.

(3)

b/ $0.5\left(\frac{0}{2} + 0.530 + 1.414 + 3.137 + \frac{6}{2}\right) = 4.0405$ units^2

$= 4.04$ (3sf)

c/ Area of triangle $= \frac{1}{2} \times 2 \times 6 = 6$ units^2

$6 - 4.0405$

$= 1.96$ units^2 3sf

5.

$$y = \sqrt{(5^x + 2)}$$

(*a*) Copy and complete the table below, giving the values of *y* to 3 decimal places.

x	0	0.5	1	1.5	2
y	1.732	2.058	2.646	3.630	5.196

(2)

(*b*) Use the trapezium rule, with all the values of *y* from your table, to find an approximation for the value of $\int_0^2 \sqrt{(5^x + 2)}\ dx$.

(4)

$0.5\left(\frac{1.732}{2} + 2.058 + 2.646 + 3.630 + \frac{5.196}{2}\right)$

$= 5.899$ units^2

6.

$$y = \sqrt{(10x - x^2)}.$$

(*a*) Copy and complete the table below, giving the values of *y* to 2 decimal places.

x	1	1.4	1.8	2.2	2.6	3
y	3	3.47	3.84	4.14	4.39	4.58

(2)

(*b*) Use the trapezium rule, with all the values of *y* from your table, to find an approximation for the value of $\int_1^3 \sqrt{(10x - x^2)}\ dx$.

(4)

$0.4\left(\frac{3}{2} + 3.47 + 3.84 + 4.14 + 4.39 + \frac{4.58}{2}\right)$

$= 7.852$ units^2

7. (*a*) Complete the table below, giving values of $\sqrt{(2^x + 1)}$ to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{(2^x+1)}$	1.414	1.554	1.732	1.957	2.236	2.580	3

(2)

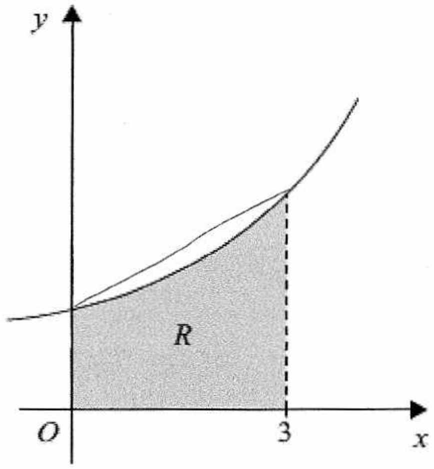

Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \sqrt{(2^x + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(*b*) Use the trapezium rule, with all the values from your table, to find an approximation for the area of R.

(4)

(*c*) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (*b*) is an overestimate or an underestimate for the area of R.

(2)

$$0.5\left(\frac{1.414}{2} + 1.554 + 1.732 + 1.957 + 2.236 + 2.580 + \frac{3}{2}\right)$$

$$= 6.133 \text{ units}^2$$

c/ it is an overestimate - the graph is curving upwards

The trapezia would go over the curve