

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Binomial Expansion

Materials required for examination
Mathematical Formulae (Pink or Green)

Items included with question papers
Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

3. (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of

$$(2 + kx)^7$$

where k is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of x ,

- (b) find the value of k .

(2)

a)

$$1 \quad 7 \quad 21$$

$$1(2)^7 + 7(2)^6(kx) + 21(2)^5(kx)^2$$

$$128 + 448kx + 672k^2x^2$$

b/

$$672k^2 = 6 \times 448k$$

$$672k^2 = 2688k$$

$$k^2 = 4k$$

$$k^2 - 4k = 0$$

$$k(k - 4) = 0$$

$k = 4$

4. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of $(1 + ax)^{10}$, where a is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^3 is double the coefficient of x^2 ,

- (b) find the value of a .

(2)

a)

$$1 \quad 10 \quad 45 \quad 120$$

$$1(1)^{10} + 10(1)^9(ax) + 45(1)^8(ax)^2 + 120(1)^7(ax)^3$$

$$1 + 10ax + 45a^2x^2 + 120a^3x^3$$

b/

$$120a^3 = 2 \times 45a^2$$

$$120a^3 = 90a^2$$

$$4a^3 = 3a^2$$

$$4a^3 - 3a^2 = 0$$

$$a^2(4a - 3) = 0$$

$$a = 0 \quad a = \frac{3}{4}$$

3

Turn over

a is non zero. $a = \frac{3}{4}$

5. (a) Find the first 4 terms of the expansion of $\left(1 + \frac{x}{2}\right)^{10}$ in ascending powers of x , giving each term in its simplest form. (4)

- (b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places. (3)

$$1 \quad 10 \quad 45 \quad 120$$

$$1(1)^{10} + 10(1)^9\left(\frac{x}{2}\right) + 45(1)^8\left(\frac{x}{2}\right)^2 + 120(1)^7\left(\frac{x}{2}\right)^3$$

$$1 + 5x + \frac{45}{4}x^2 + 15x^3$$

b/ $1 + \frac{x}{2} = 1.005$
 $\frac{x}{2} = 0.005$
 $x = 0.01$

$$1 + 5(0.01) + \frac{45}{4}(0.01)^2 + 15(0.01)^3$$

$$= \underline{\underline{1.05114}}$$

6. (a) Find the first four terms, in ascending powers of x , in the binomial expansion of $(1 + kx)^6$, where k is a non-zero constant. (3)

Given that, in this expansion, the coefficients of x and x^2 are equal, find

- (b) the value of k , (2)

- (c) the coefficient of x^3 . (1)

$$1 \quad 6 \quad 15 \quad 20$$

a/ $1(1)^6 + 6(1)^5(kx) + 15(1)^4(kx)^2 + 20(1)^3(kx)^3$
 $1 + 6kx + 15k^2x^2 + 20k^3x^3$

b/ $6k = 15k^2$
 $2k = 5k^2$
 $0 = 5k^2 - 2k$
 $0 = k(5k - 2)$
 $k = 0 \quad \underline{\underline{k = 2/5}} \quad k \text{ is non zero } \therefore k = 2/5$

c/ $20\left(\frac{2}{5}\right)^3 = \underline{\underline{\frac{32}{25}}}$

9. (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of

$$(1 + px)^9,$$

where p is a constant.

(2)

The first 3 terms are 1 , $36x$ and qx^2 , where q is a constant.

- (b) Find the value of p and the value of q .

(4)

$$\begin{array}{cccc} & & 1 & 9 & 36 \\ a) & 1(1)^9 & + & 9(1)^8(px) & + & 36(1)^7(px)^2 \\ & 1 & + & 9px & + & 36p^2x^2 \end{array}$$

$$\begin{array}{l} b/ \quad 36 = 9p \\ \quad \quad \underline{p = 4} \\ \quad \quad 36p^2 = q \\ \quad \quad 36(4)^2 = q \\ \quad \quad \underline{q = 576} \end{array}$$

10. (a) Write down the first three terms, in ascending powers of x , of the binomial expansion of $(1 + px)^{12}$, where p is a non-zero constant.

(2)

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of x is $-q$ and the coefficient of x^2 is $11q$,

- (b) find the value of p and the value of q .

(4)

$$\begin{array}{cccc} & & 1 & 12 & 66 \\ a) & 1(1)^{12} & + & 12(1)^{11}(px) & + & 66(1)^{10}(px)^2 \\ & 1 & + & 12px & + & 66p^2x^2 \end{array}$$

$$\begin{array}{l} b/ \quad 12p = -q \\ \quad \quad 66p^2 = 11q \\ \quad \quad 6p^2 = q \\ \quad \quad 6p^2 = -12p \end{array}$$

$$6p^2 + 12p = 0$$

$$6p(p + 2) = 0$$

$$p = 0 \quad p = -2$$

$$p \text{ is non zero } \therefore \underline{p = -2}$$

$$\begin{array}{l} q = -12p \\ = -12(-2) \\ = \underline{\underline{24}} \end{array}$$