C1 : Algebra and Functions

1a) $4x - 3 > 7 - x$

$5x - 3 > 7$

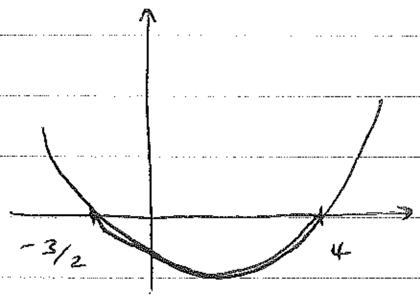
$5x > 10$

$x > 2$

b/ $2x^2 - 5x - 12 < 0$

$(2x+3)(x-4) < 0$

$x = -3/2 \quad x = 4$



$\underline{-\tfrac{3}{2} < x < 4}$

c/ $2 < x < 4$

2/ $x^2 + 3px + p = 0$

$a = 1 \quad b = 3p \quad c = p$

equal roots so $b^2 - 4ac = 0$

$(3p)^2 - 4(1)(p) = 0$

$9p^2 - 4p = 0$

$p(9p - 4) = 0$

$p = 0 \quad p = 4/9$

$\therefore p = 4/9$

3/ $x^3 - 9x$

$x(x^2 - 9)$

$x(x+3)(x-3)$

4a) $y = x - 4$

$2x^2 - xy = 8$

$2x^2 - x(x-4) = 8$

$2x^2 - x^2 + 4x = 8$

$x^2 + 4x - 8 = 0$

$(x+2)^2 - 12 = 0$

$(x+2)^2 = 12$

$x + 2 = \pm\sqrt{12}$

$x = -2 \pm 2\sqrt{3}$

$y = -6 \pm 2\sqrt{3}$

5a) $x^2 - 8x - 29$

$(x-4)^2 - 45$

$a = -4 \quad b = -45$

b) $x^2 - 8x - 29 = 0$

$(x-4)^2 - 45 = 0$

$(x-4)^2 = 45$

$x - 4 = \pm\sqrt{45}$

$x = 4 \pm \sqrt{45}$

$= \underline{4 \pm 3\sqrt{5}}$

6/ $y = x - 2$

$y^2 + x^2 = 10$

$(x-2)^2 + x^2 = 10$

$x^2 - 4x + 4 + x^2 = 10$

$2x^2 - 4x - 6 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

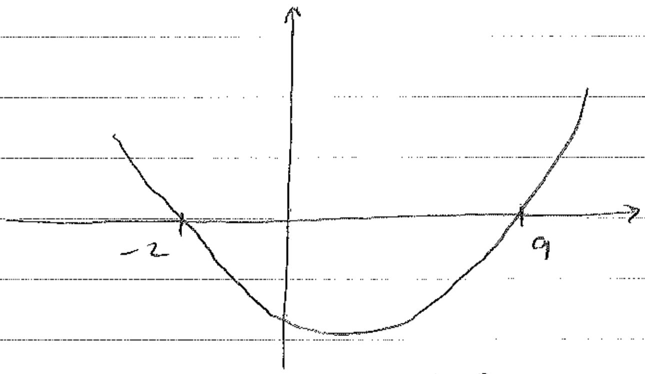
$x = 3 \quad x = -1$

$y = 1 \quad y = -3$

7/ $x^2 - 7x - 18 > 0$

$(x-9)(x+2) > 0$

$x = 9 \quad x = -2$



$\underline{x < -2} \qquad \underline{x > 9}$

8/ $x^3 - 4x^2 + 3x$

$x(x^2 - 4x + 3)$

$x(x-3)(x-1)$

9/ $kx^2 + 4x + (5-k) = 0$

$a = k \quad b = 4 \quad c = (5-k)$

2 Solutions $\therefore$ $b^2 - 4ac > 0$

$(4)^2 - 4(k)(5-k) > 0$

$16 - 4k(5-k) > 0$

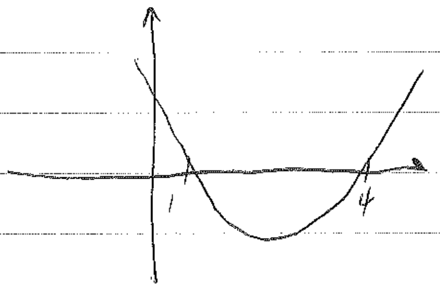
$16 - 20k + 4k^2 > 0$

$4k^2 - 20k + 16 > 0$

$k^2 - 5k + 4 > 0$

b/ $(k-4)(k-1) > 0$

$k = 4 \quad k = 1$



$k < 1$ or $k > 4$

10/ $x - 2y = 1$ $\quad x = 1 + 2y$

$x^2 + y^2 = 29$

$(1+2y)^2 + y^2 = 29$

$1 + 4y + 4y^2 + y^2 = 29$

$5y^2 + 4y - 28 = 0$

$(5y+14)(y-2) = 0$

$y = -\frac{14}{5} \quad y = 2$

$x = -\frac{23}{5} \quad x = 5$

11/ $2qx^2 + qx - 1 = 0$

$a = 2q \quad b = q \quad c = -1$

no roots $\therefore$ $b^2 - 4ac < 0$

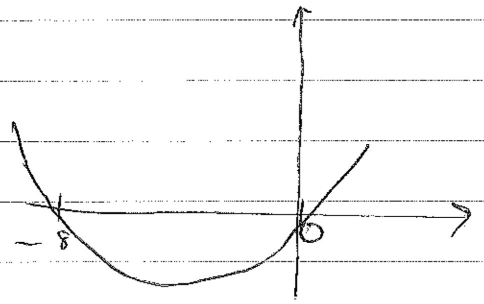
$(q)^2 - 4(2q)(-1) < 0$

$q^2 + 4(2q) < 0$

$q^2 + 8q < 0$

b/ $q(q+8) < 0$

$q = 0 \quad q = -8$



$\underline{-8 < q < 0}$

13a) $x^2 + kx + (k+3) = 0$

different roots $\therefore$ $b^2 - 4ac > 0$

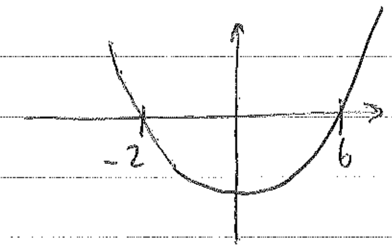
$a = 1 \quad b = k \quad c = (k+3)$ $\quad (k)^2 - 4(1)(k+3) > 0$

$k^2 - 4k - 12 > 0$

b/ $k^2 - 4k - 12 > 0$

$(k-6)(k+2) > 0$

$k = 6 \quad k = -2$



$k < -2$ or $k > 6$

14/ $x + y = 2 \qquad x = 2 - y$

$x^2 + 2y = 12$

$(2-y)^2 + 2y = 12$

$4 - 4y + y^2 + 2y = 12$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y = 4 \quad y = -2$

$x = -2 \quad x = 4$

15/ $2x^2 - 3x - (k+1) = 0$

$a = 2 \quad b = -3 \quad c = -(k+1)$

no real roots $\therefore b^2 - 4ac < 0$

$(-3)^2 - 4(2)(-k-1) < 0$

$9 - 8(-k-1) < 0$

$9 + 8k + 8 < 0$

$8k < -17$

$k < -17/8$

16/ $x^2 + 2px + (3p+4) = 0$

$a = 1 \quad b = 2p \quad c = 3p+4 \qquad b^2 - 4ac = 0$

$(2p)^2 - 4(1)(3p+4) = 0$

$4p^2 - 12p - 16 = 0$

$\underline{p = 4} \qquad p^2 - 3p - 4 = 0$

$(p-4)(p+1) = 0$

b/ $p = 4$

$$x^2 + 2(4)x + (3(4) + 4) = 0$$

$$x^2 + 8x + 16 = 0$$

$$(x+4)(x+4) = 0$$

$$\underline{\underline{x = -4}}$$

17a/ $3(2x+1) > 5 - 2x$

$$6x + 3 > 5 - 2x$$

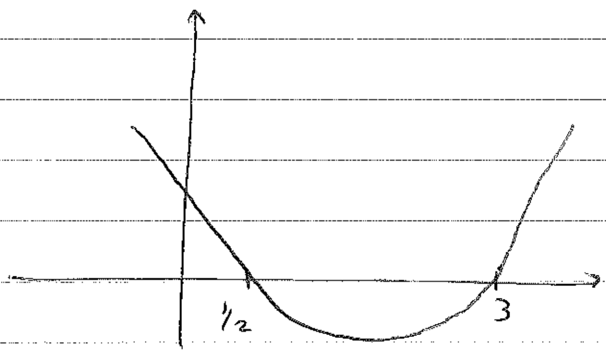
$$8x > 2$$

$$x > 1/4$$

b/ $2x^2 - 7x + 3 > 0$

$$(2x-1)(x-3) > 0$$

$$x = 1/2 \qquad x = 3$$



$\underline{x < 1/2}$ or $\underline{x > 3}$

c/ $1/4 < x < 1/2$ or $x > 3$

18/ $kx^2 + 12x + k = 0$

$a = k \quad b = 12 \quad c = k$

equal roots $\therefore$ $b^2 - 4ac = 0$

$$(12)^2 - 4(k)(k) = 0$$

$$144 - 4k^2 = 0$$

$$144 = 4k^2$$

$$36 = k^2$$

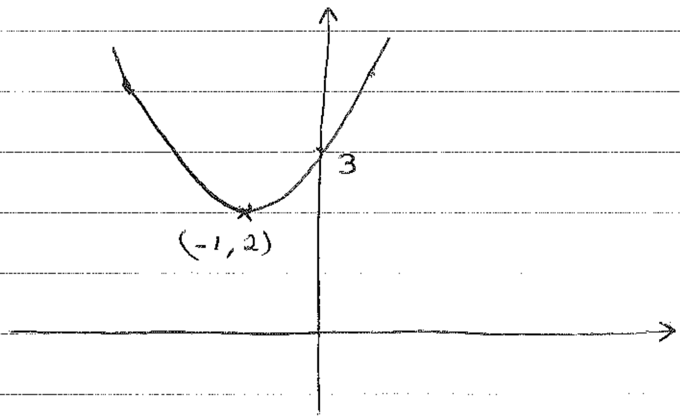
$$\underline{\underline{k = 6}}$$

19a/ $x^2 + 2x + 3$

$(x+1)^2 + 2$

$a = 1 \quad b = 2$

b/ $y = x^2 + 2x + 3$



c) $b^2 - 4ac$

$(2)^2 - 4(1)(3)$

$4 - 12$

-8

as $b^2 - 4ac < 0$ there are no solutions to $x^2 + 2x + 3 = 0$

it does not cross the x axis

d/ $x^2 + kx + 3 = 0$

$a = 1 \quad b = k \quad c = 3$

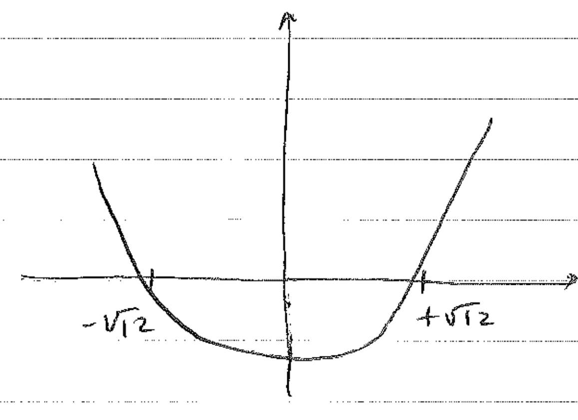
$b^2 - 4ac < 0$

$k^2 - 4(1)(3) < 0$

$k^2 - 12 < 0$

$k^2 = 12 \quad \cancel{k^2 < 12}$

$k = \pm\sqrt{12}$



$-\sqrt{12} < k < \sqrt{12}$

$\underline{-2\sqrt{3} < k < 2\sqrt{3}}$