Name: ______________________________

Maths Genie Stage 14

Test D

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- **Calculators may be used.**

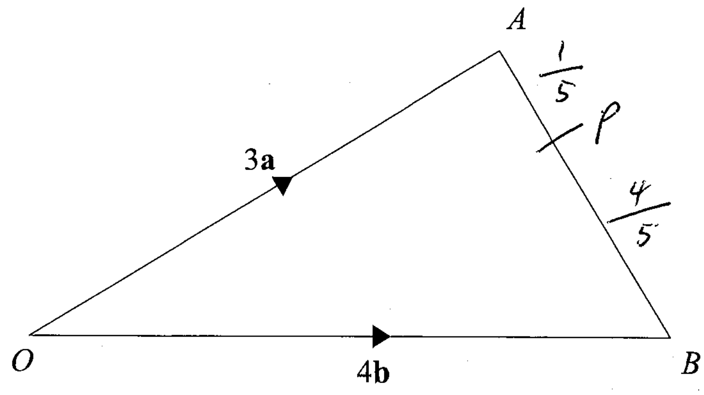
Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1



$\overrightarrow{OA} = 3a$

$\overrightarrow{OB} = 4b$

P is the point on AB such that AP:PB = 1:4

$\overrightarrow{OP} = k(3\boldsymbol{a} + \boldsymbol{b})$

Find the value of k

$$\overrightarrow{AB} = -3a + 4b$$

$$\overrightarrow{AP} = \frac{1}{5}(-3a + 4b)$$

$$= -\frac{3}{5}a + \frac{4}{5}b$$

$$\overrightarrow{OP} = 3a - \frac{3}{5}a + \frac{4}{5}b$$

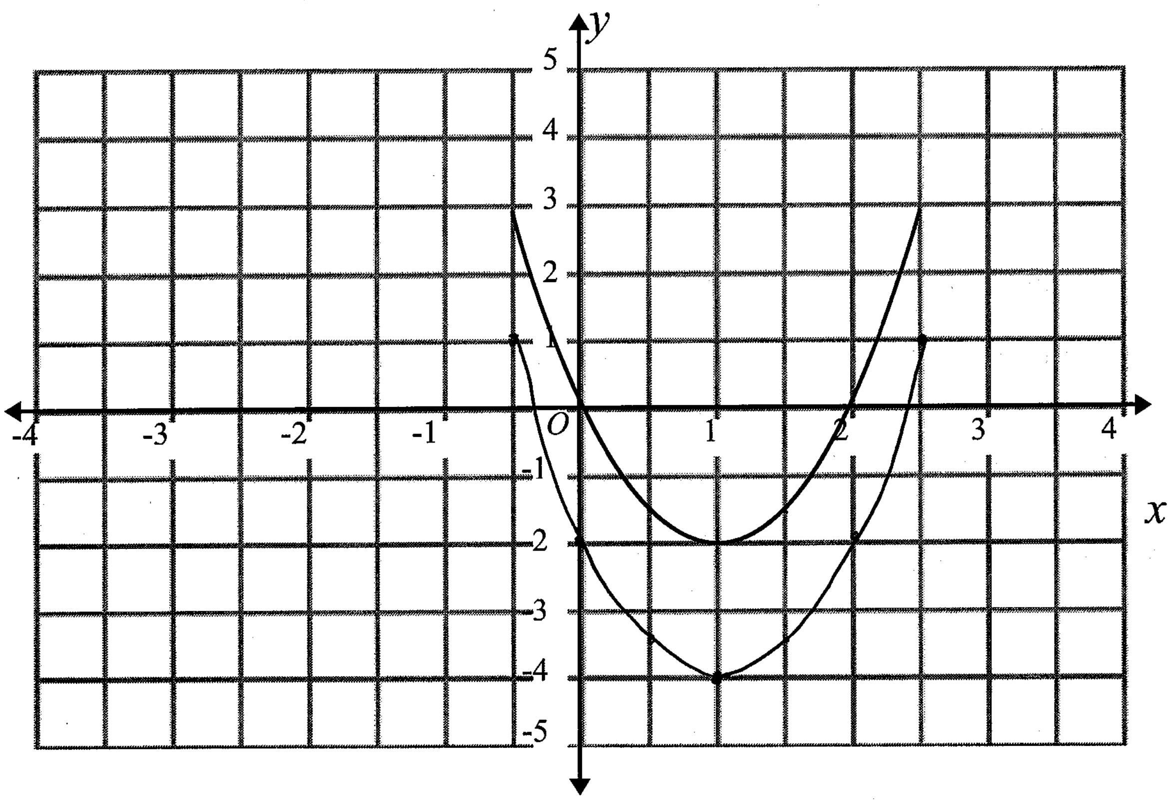
$$= \frac{12}{5}a + \frac{4}{5}b$$

$$= \frac{4}{5}(3a + b)$$

$k = \frac{4}{5}$ ....

(Total for Question 1 is 3 marks)

2 The graph of $y = f(x)$ is shown on the grid.



(a) On the grid above, sketch the graph of $y = f(x) - 2$

(1)

The graph of $y = f(x)$ has a turning point at $(1, -2)$.

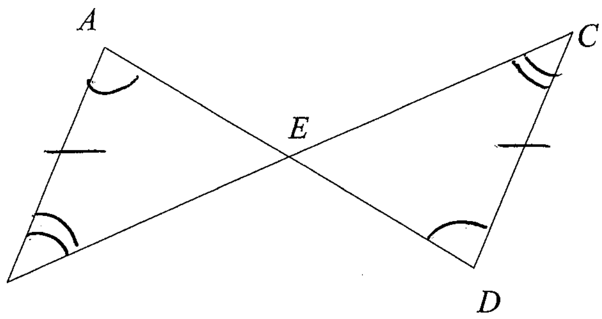
(b) Write down the coordinates of the turning point of $y = -f(x + 3)$

$(-2, 2)$

(1)

(Total for Question 2 is 2 marks)

3



AB and CD are parallel and equal in length.

Prove that triangle ABE and triangle CDE are congruent.

$AB = CD$ (Given)

$BAE = CDE$ alternate angles are equal

$ABE = DCE$ ——— " ———

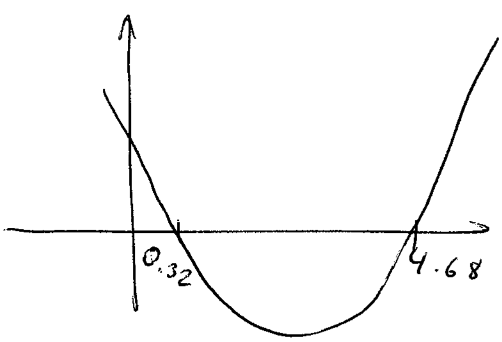
ASA

(Total for Question 3 is 3 marks)

4 Work out the integer values that satisfy: $2x^2 - 10x + 3 < 0$

$a = 2$
$b = -10$
$c = 3$

Roots at $x = 4.68$ and 0.32



1, 2, 3 and 4

(Total for Question 4 is 4 marks)

5 Solve the simultaneous equations

$$x^2 + y^2 = 29$$
$$y = 2x - 1$$

$$x^2 + (2x-1)^2 = 29$$

$$x^2 + (2x-1)(2x-1) = 29$$

$$x^2 + 4x^2 - 2x - 2x + 1 = 29$$

$$5x^2 - 4x + 1 = 29$$

$$5x^2 - 4x - 28 = 0$$

$$(5x-14)(x+2) = 0$$

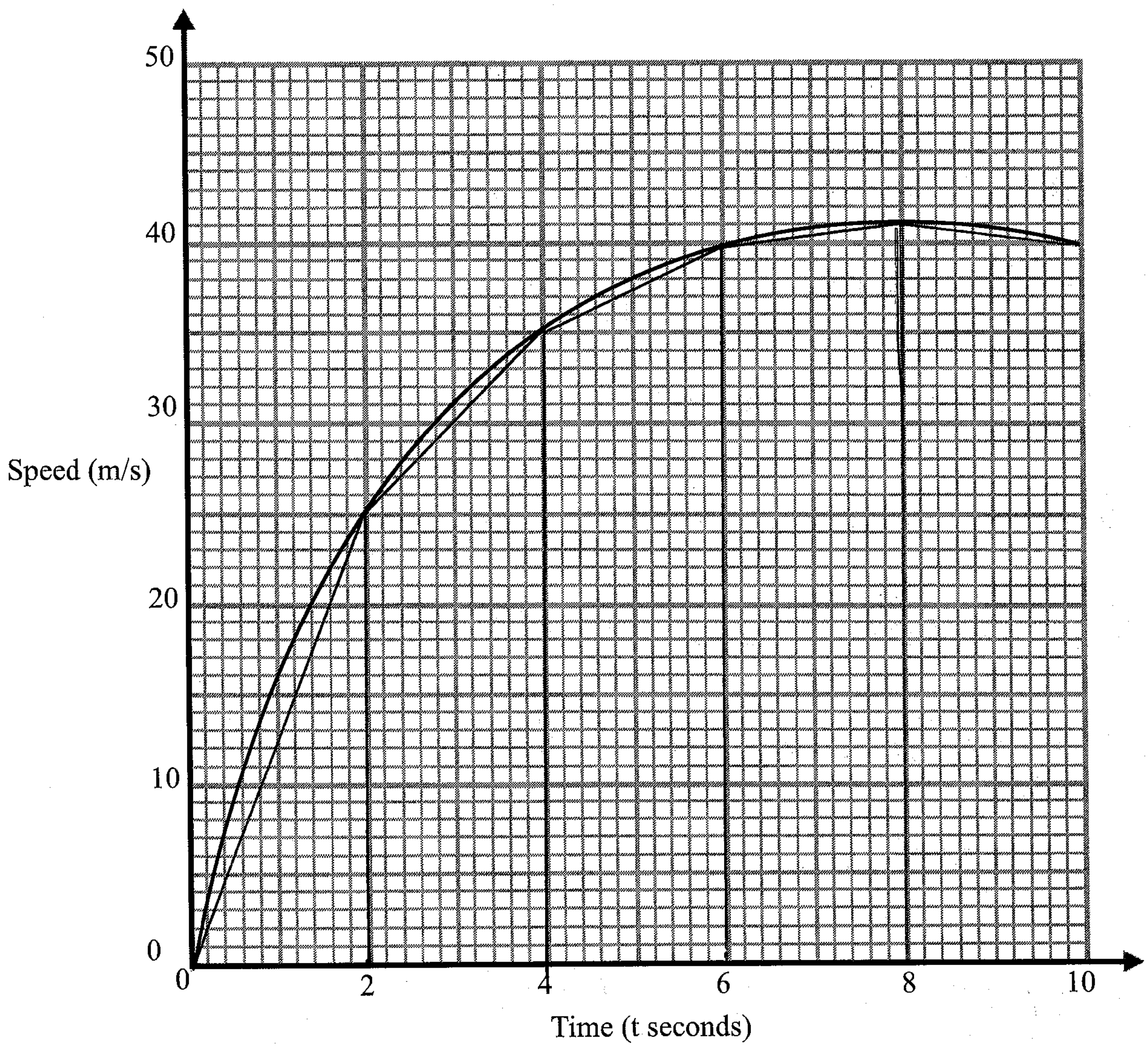
$$x = \frac{14}{5} \qquad x = -2$$

$$y = 2\left(\frac{14}{5}\right) - 1 \qquad y = 2(-2) - 1$$
$$= \frac{23}{5} \qquad = -5$$

$$x = \frac{14}{5},\ y = \frac{23}{5} \quad \text{or} \quad x = -2,\ y = -5$$

(Total for Question 5 is 5 marks)

6 Here is a speed-time graph.



Use 5 strips of equal width to find an estimate for the distance travelled in 10 seconds.

$\frac{1}{2}(2)(25) = 25$

$\frac{1}{2}(25 + 35) \times 2 = 60$

$\frac{1}{2}(35 + 40) \times 2 = 75$

$\frac{1}{2}(40 + 41) \times 2 = 81$

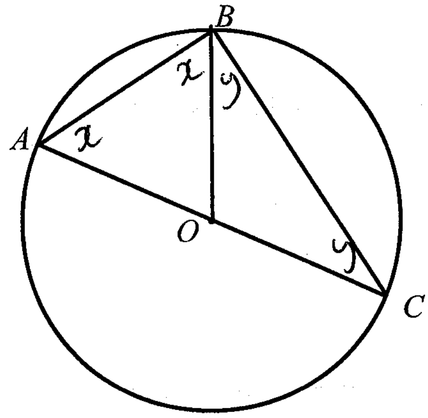
$\frac{1}{2}(41 + 40) \times 2 = 81$

$$\begin{array}{r} 81 \\ 81 \\ 75 \\ 60 \\ +\ 25 \\ \hline 322 \end{array}$$

322 m

(Total for Question 6 is 3 marks)

7



A, B and C are points on the circumference of a circle, centre O.
AOC is a diameter of the circle.

Prove that angle ABC is 90°
You must **not** use any circle theorems in your proof.

Let $OAB = x$ $\qquad ABC = x + y$
Let $OBC = y$

$OBA = x$ Angles at the base of an isosceles triangle are equal.
$OCB = y$

$AOB = 180 - 2x$ Angles in a triangle add to 180°
$BOC = 180 - 2y$

$180 - 2x + 180 - 2y = 180$ Angles on a straight line add to 180°

$360 - 2x - 2y = 180$

$180 = 2x + 2y$

$\underline{\underline{90 = x + y}}$

$\underline{\underline{90 = ABC}}$

(Total for Question 7 is 4 marks)

8 A circle has the equation $x^2 + y^2 = 17$

(a) Write down the coordinates of the centre of the circle.

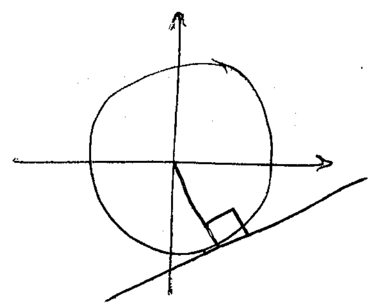
$(0, 0)$

(1)

P is the point (1,-4) on the circle $x^2 + y^2 = 17$

(b) Work out the equation of the tangent to the circle at P.

Radius goes through $\overset{x_1\ y_1}{(0, 0)}$ and $\underset{x_2\ y_2}{(1, -4)}$



Gradient of radius $= \dfrac{-4-0}{1-0}$

$= -4$

Perp gradient $= \dfrac{1}{4}$

$y = \dfrac{1}{4}x + c$

$-4 = \dfrac{1}{4}(1) + c$

$-4 = \dfrac{1}{4} + c$

$-16 = 1 + 4c$

$-17 = 4c$

$c = \dfrac{-17}{4}$

$y = \dfrac{1}{4}x - \dfrac{17}{4}$

(4)

(Total for Question 8 is 5 marks)

9 There are n counters in a bag.

$n-5$ Blue.

5 of the counters are red and the rest are blue.

Ross takes a counter from the bag at random and does not replace it.
He then takes another counter at random from the bag.

The probability that Ross takes two blue counters is $\frac{3}{7}$

Find the value of n.

$$P(\text{Blue, Blue}) = \frac{n-5}{n} \times \frac{n-6}{n-1}$$

$$\frac{n-5}{n} \times \frac{n-6}{n-1} = \frac{3}{7}$$

$$\frac{(n-5)(n-6)}{n(n-1)} = \frac{3}{7}$$

$$7(n-5)(n-6) = 3n(n-1)$$

$$7(n^2 - 6n - 5n + 30) = 3n^2 - 3n$$

$$7n^2 - 77n + 210 = 3n^2 - 3n$$

$$4n^2 - 74n + 210 = 0$$

$$2n^2 - 37n + 105 = 0$$

$$(2n-7)(n-15) = 0$$

$$n = \frac{7}{2} \qquad \underline{\underline{n = 15}}$$

n ~~cannot~~ be ~~a~~
n must be an integer

.............15.............

(Total for Question 9 is 6 marks)