

**1.** Find



giving your answer in simplest form.

**(4)**

**(Total for Question 1 is 4 marks)**

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**2.** f(*x*) = 2*x*3 + 5*x*2 + 2*x* + 15

(*a*)Use the factor theorem to show that (*x* + 3) is a factor of f(*x*).

**(2)**

(*b*)Find the constants *a*, *b* and *c* such that

f(*x*) = (*x* + 3)(*ax*2 + *bx* + *c*)

**(2)**

(*c*)Hence show that f(*x*) = 0 has only one real root.

**(2)**

(*d*)Write down the real root of the equation f(*x* – 5) = 0

**(1)**

**(Total for Question 2 is 7 marks)**

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**3.** The triangle *PQR* is such that  = 3**i** + 5**j** and = 13**i** − 15**j**

(*a*)Find 

**(2)**

(*b*)Hence find  giving your answer as a simplified surd.

**(2)**

The point *S* lies on the line segment *QR* so that *QS* : *SR* = 3 : 2

(*c*)Find 

**(2)**

**(Total for Question 3 is 6 marks)**

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**4.**

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**Figure 1**

Figure 1 shows a sketch of triangle *ABC* with *AB* = (*x* + 2) cm, *BC* = (3*x* + 10) cm,

*AC* = 7*x* cm, angle *BAC* = 60° and angle *ACB* = *θ*°

(*a*)(i) Show that 17*x*2 – 35*x* – 48 = 0

**(3)**

(ii) Hence find the value of *x.*

**(1)**

(*b*)Hence find the value of *θ* giving your answer to one decimal place.

**(2)**

**(Total for Question 4 is 6 marks)**

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**5.** The mass, *A* kg, of algae in a small pond, is modelled by the equation

*A* = *pqt*

where *p* and *q* are constants and *t* is the number of weeks after the mass of algae was

first recorded.

Data recorded indicates that there is a linear relationship between *t* and log10 *A* given by

the equation

log10 *A* = 0.03*t* + 0.5

(*a*)Use this relationship to find a complete equation for the model in the form

*A* = *pqt*

giving the value of *p* and the value of *q* each to 4 significant figures.

**(4)**

(*b*)With reference to the model, interpret

(i) the value of the constant *p*,

(ii) the value of the constant *q*.

**(2)**

(*c*)Find, according to the model,

(i) the mass of algae in the pond when *t* = 8, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

**(3)**

(*d*)State one reason why this may not be a realistic model in the long term.

**(1)**

**(Total for Question 5 is 10 marks)**

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**6.** (*a*)Find the first 4 terms, in ascending powers of *x*, of the binomial expansion of



giving each term in simplest form.



**(4)**

(*b*)Find the coefficient of *x*2 in the series expansion of f(*x*), giving your answer as a

simplified fraction.

**(2)**

**(Total for Question 6 is 6 marks)**

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**7.** (*a*)Factorise completely 9*x* – *x*3

**(2)**

The curve *C* has equation

*y* = 9*x* – *x*3

(*b*)Sketch *C* showing the coordinates of the points at which the curve cuts the *x*‑axis.

**(2)**

The line *l* has equation *y* = *k* where *k* is a constant.

Given that *C* and *l* intersect at 3 distinct points,

(*c*)find the range of values for *k*, writing your answer in set notation.

**Solutions relying on calculator technology are not acceptable.**

**(3)**

**(Total for Question 7 is 7 marks)**

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**8. In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The air pressure, *P* kg/cm2, inside a car tyre, *t* minutes from the instant when the tyre

developed a puncture is given by the equation

*P* = *k* + 1.4e–0.5*t t* ∈ ℝ *t* ≥ 0

where *k* is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm2

(*a*)state the value of *k*.

**(1)**

From the instant when the tyre developed the puncture,

(*b*)find the time taken for the air pressure to fall to 1 kg/cm2

Give your answer in minutes to one decimal place.

**(3)**

(*c*)Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes

from the instant when the tyre developed the puncture.

Give your answer in kg/cm2 per minute to 3 significant figures.

**(2)**

**(Total for Question 8 is 6 marks)**

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**9.** (*a*)Given that *p* = log3 *x*, where *x* > 0, find in simplest form in terms of *p*,

(i) 

(ii) 

**(2)**

(*b*)Hence, or otherwise, solve



giving your answer as a simplified fraction.

**Solutions relying on calculator technology are not acceptable.**

**(4)**

**(Total for Question 9 is 6 marks)**

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**10.**

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**Figure 2**

**In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Figure 2 shows a sketch of part of the curve *C* with equation

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The point *P* lies on *C* and has *x* coordinate 4

The line *l* is the tangent to *C* at *P*.

(*a*)Show that *l* has equation

13*x* – 6*y* – 26 = 0

**(5)**

The region *R*, shown shaded in Figure 2, is bounded by the *y*‑axis, the curve *C*, the line *l*

and the *x*‑axis.

(*b*)Find the exact area of *R*.

**(5)**

**(Total for Question 10 is 10 marks)**

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**11.**

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**Figure 3**

Figure 3 shows the circle *C* with equation

*x*2 + *y*2 – 10*x* – 8*y* + 32 = 0

and the line *l* with equation

2*y* + *x* + 6 = 0

(*a*)Find

(i) the coordinates of the centre of *C*,

(ii) the radius of *C*.

**(3)**

(*b*)Find the shortest distance between *C* and *l*.

**(5)**

**(Total for Question 11 is 8 marks)**

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**12.** A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius *r* cm and height *h* cm and

the capacity of each container is 355 cm3

The metal used

• for the circular base and the curved side costs 0.04 pence/cm2

• for the circular top costs 0.09 pence/cm2

Both metals used are of negligible thickness.

(*a*)Show that the total cost, *C* pence, of the metal for one container is given by

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**(4)**

(*b*)Use calculus to find the value of *r* for which *C* is a minimum, giving your answer to

3 significant figures.

**(4)**

(*c*)Using  prove that the cost is minimised for the value of *r* found in part (*b*).

**(2)**

(*d*)Hence find the minimum value of *C*, giving your answer to the nearest integer.

**(2)**

**(Total for Question 12 is 12 marks)**

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**13. In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(*a*)Show that



**(3)**

Given that cos 2*x* ≠ 0

(*b*)solve for 0 < *x* < 90°



giving your answers to one decimal place.

**(5)**

**(Total for Question 13 is 8 marks)**

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**14.** (i) A student states

“if *x*2 is greater than 9 then *x* must be greater than 3”

Determine whether or not this statement is true, giving a reason for your answer.

**(1)**

(ii) Prove that for all positive integers *n*,

*n*3 + 3*n*2 + 2*n*

is divisible by 6

**(3)**

**(Total for Question 14 is 4 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**