

**1. In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

*Diagram

Description automatically generated*

Figure 1 shows a sketch of the graph with equation *y* = | 3 − 2*x* |

Solve

| 3 − 2*x* | = 7 + *x*

**(4)**

**(Total for Question 1 is 4 marks)**

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**2.** (*a*)Sketch the curve with equation

*y* = 4*x*

stating any points of intersection with the coordinate axes.

**(2)**

(*b*)Solve

4*x* = 100

giving your answer to 2 decimal places.

**(2)**

**(Total for Question 2 is 4 marks)**

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**3.** A sequence of terms *a*1, *a*2, *a*3, … is defined by

*a*1 = 3

*an*+1 = 8 − *an*

(*a*)(i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

**(2)**

(*b*)Find the value of



**(2)**

**(Total for Question 3 is 4 marks)**

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**4.** Given that

*y* = 2*x*2

use differentiation from first principles to show that

= 4*x*

**(3)**

**(Total for Question 4 is 3 marks)**

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**5.** The table below shows corresponding values of *x* and *y* for *y* = log3 2*x*

The values of *y* are given to 2 decimal places as appropriate.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x* | 3 | 4.5 | 6 | 7.5 | 9 |
| *y* | 1.63 | 2 | 2.26 | 2.46 | 2.63 |

(*a*)Using the trapezium rule with all the values of *y* in the table, find an estimate for

****

**(3)**

Using your answer to part (*a*)and making your method clear, estimate

(*b*) (i) ****

(ii) ****

**(3)**

**(Total for Question 5 is 6 marks)**

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**6.**

Diagram

Description automatically generated

Figure 2 shows a sketch of part of the curve with equation *y* = f(*x*) where

f(*x*) = 8 sin

and *x* is measured in radians.

The point *P*, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(*a*)find the *x* coordinate of *P*, giving your answer to 3 significant figures.

**(4)**

The curve crosses the *x*-axis at *x* = *α*, as shown in Figure 2.

Given that, to 3 decimal places, f(4) = 4.274 and f(5) = −1.212

(*b*)explain why *α* must lie in the interval [4, 5]

**(1)**

(*c*)Taking *x*0 = 5 as a first approximation to *α*, apply the Newton-Raphson method once

to f(*x*) to obtain a second approximation to *α*.

Show your method and give your answer to 3 significant figures.

**(2)**

**(Total for Question 6 is 7 marks)**

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**7.** (*a*)Find the first four terms, in ascending powers of *x*, of the binomial expansion of



writing each term in simplest form.

**(4)**

A student uses this expansion with *x* =  to find an approximation for 

Using the answer to part (*a*)and without doing any calculations,

(*b*)state whether this approximation will be an overestimate or an underestimate of 3

giving a brief reason for your answer.

**(1)**

**(Total for Question 7 is 5 marks)**

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**8. In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Diagram

Description automatically generated

Figure 3 shows a sketch of part of a curve with equation



The region *R*, shown shaded in Figure 3, is bounded by the curve and the *x*-axis.

Find the exact area of *R*, writing your answer in the form **, where *a* and *b* are

constants to be found.

**(6)**

**(Total for Question 8 is 6 marks)**

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**9.**

*Diagram, shape

Description automatically generated*

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, *H* m, of a passenger on the Ferris wheel, *t* seconds after

the wheel starts turning, is modelled by the equation

*H* = |*A* sin(*bt* + α)°|

where *A*, *b* and α are constants.

Figure 5 shows a sketch of the graph of *H* against *t*, for one revolution of the wheel.

Given that

• the maximum height of the passenger above the ground is 50 m

• the passenger is 1 m above the ground when the wheel starts turning

• the wheel takes 720 seconds to complete one revolution

(*a*)find a complete equation for the model, giving the exact value of *A*, the exact value

of *b* and the value of α to 3 significant figures.

**(4)**

(*b*)Explain why an equation of the form

*H* = |*A* sin(*bt* + α)°| + *d*

where *d* is a positive constant, would be a more appropriate model.

**(1)**

**(Total for Question 9 is 5 marks)**

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**10.** The function f is defined by

f(*x*) = 

(*a*)Find f −1

**(2)**

(*b*)Show that

f(*x*) = *A* + 

where *A* and *B* are constants to be found.

**(2)**

The function g is defined by

g(*x*) = 16 − *x*2 0 ≤ *x* ≤ 4

(*c*)State the range of g−1

**(1)**

(*d*)Find the range of f g−1

**(3)**

**(Total for Question 10 is 8 marks)**

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**11.** Prove, using algebra, that

*n*(*n*2 + 5)

is even for all *n* ∈ ℕ

**(4)**

**(Total for Question 11 is 4 marks)**

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**12.** The function f is defined by

f(*x*) = 

where *k* is a positive constant.

(*a*)Show that

f ʹ(*x*) = (12*x*2 − 8*x* + 3*k*) g(*x*)

where g(*x*) is a function to be found.

**(3)**

Given that the curve with equation *y* = f(*x*) has at least one stationary point,

(*b*)find the range of possible values of *k*.

**(3)**

**(Total for Question 12 is 6 marks)**

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**13.** Relative to a fixed origin *O*

• the point *A* has position vector 4**i** − 3**j** + 5**k**

• the point *B* has position vector 4**j** + 6**k**

• the point *C* has position vector −16**i** + *p***j** + 10**k**

where *p* is a constant.

Given that *A*, *B* and *C* lie on a straight line,

(*a*)find the value of *p*.

**(3)**

The line segment *OB* is extended to a point *D* so that is parallel to 

(*b*)Find ||, writing your answer as a fully simplified surd.

**(3)**

**(Total for Question 13 is 6 marks)**

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**14.** (*a*)Express  in partial fractions.

**(3)**

When chemical *A* and chemical *B* are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen

produced over a period of time.

The total volume of oxygen produced, *V* m3 , *t* hours after the chemicals were mixed, is

modelled by the differential equation

 *V* ≥ 0 *t* ≥ *k*

where *k* is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of 3 m3 of

oxygen had been produced,

(*b*)solve the differential equation to show that

*V* = 

**(5)**

The scientist noticed that

• there was a **time delay** between the chemicals being mixed and oxygen

being produced

• there was a **limit** to the total volume of oxygen produced

Deduce from the model

(*c*)(i) the **time delay** giving your answer in minutes,

(ii) the **limit** giving your answer in m3

**(2)**

**(Total for Question 14 is 10 marks)**

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**15. In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Given that the first three terms of a geometric series are

12 cos *θ* 5 + 2 sin *θ* and 6 tan *θ*

(*a*)show that

4 sin2 *θ* − 52 sin *θ* + 25 = 0

**(3)**

Given that *θ* is an obtuse angle measured in radians,

(*b*)solve the equation in part (*a*)to find the exact value of *θ*

**(2)**

(*c*)show that the sum to infinity of the series can be expressed in the form

*k* (1 − )

where *k* is a constant to be found.

**(5)**

**(Total for Question 15 is 10 marks)**

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**16.**

Diagram

Description automatically generated

Figure 6 shows a sketch of the curve *C* with parametric equations

*x* = 2 tan *t* + 1 *y* = 2 sec2 *t* + 3 −

The line *l* is the normal to *C* at the point *P* where *t* = 

(*a*)Using parametric differentiation, show that an equation for *l* is

*y* = 

**(5)**

(*b*)Show that all points on *C* satisfy the equation

*y* = 

**(2)**

The straight line with equation

*y* = *x* + *k* where *k* is a constant

intersects *C* at two distinct points.

(*c*)Find the range of possible values for *k*.

**(5)**

**(Total for Question 16 is 12 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**