****

**1.** The point *P* (−2, −5) lies on the curve with equation *y* = f (*x*), *x* ∈ ℝ

Find the point to which *P* is mapped, when the curve with equation *y* = f (*x*)

is transformed to the curve with equation

(*a*) *y* = f (*x*) + 2

**(1)**

(*b*) *y* = | f (*x*) |

**(1)**

(*c*) *y* = 3f (*x* − 2) + 2

**(2)**

**(Total for Question 1 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**2.** f (*x*) = (*x* − 4)(*x*2 − 3*x* + *k*) − 42 where *k* is a constant

Given that (*x* + 2) is a factor of f (*x*), find the value of *k*.

**(3)**

**(Total for Question 2 is 3 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**3.** A circle has equation

*x*2 + *y*2 − 10*x* + 16*y* = 80

(*a*)Find

 (i) the coordinates of the centre of the circle,

 (ii) the radius of the circle.

**(3)**

Given that *P* is the point on the circle that is furthest away from the origin *O*,

(*b*)find the exact length *OP*

**(2)**

**(Total for Question 3 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**4.** (*a*)Express  as an integral.

**(1)**

(*b*)Hence show that



where *k* is a constant to be found.

**(2)**

**(Total for Question 4 is 3 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**5.** The height, *h* metres, of a tree, *t* years after being planted, is modelled by the equation

*h*2 = *at* + *b* 0 ≤ *t* < 25

where *a* and *b* are constants.

Given that

• the height of the tree was 2.60 m, exactly 2 years after being planted

• the height of the tree was 5.10 m, exactly 10 years after being planted

(*a*)find a complete equation for the model, giving the values of *a* and *b* to 3 significant figures.

**(4)**

Given that the height of the tree was 7 m, exactly 20 years after being planted

(*b*)evaluate the model, giving reasons for your answer.

**(2)**

**(Total for Question 5 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**6.**



Figure 1 shows a sketch of a curve *C* with equation *y* = f (*x*) where f (*x*) is a cubic

expression in *x*.

The curve

• passes through the origin

• has a maximum turning point at (2, 8)

• has a minimum turning point at (6, 0)

(*a*)Write down the set of values of *x* for which

f ʹ(*x*) < 0

**(1)**

The line with equation *y* = *k*, where *k* is a constant, intersects *C* at only one point.

(*b*)Find the set of values of *k*, giving your answer in set notation.

**(2)**

(*c*)Find the equation of *C*. You may leave your answer in factorised form.

**(3)**

**(Total for Question 6 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**7.** (i) Given that *p* and *q* are integers such that

*pq* is even

use algebra to prove by contradiction that at least one of *p* or *q* is even.

**(3)**

(ii) Given that *x* and *y* are integers such that

* *x* < 0

• (*x* + *y*)2 < 9*x*2 + *y*2

show that *y* > 4*x*

**(2)**

**(Total for Question 7 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**8.**



A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, *v* ms−1, as it travels between the two sets

of traffic lights.

The car takes *T* seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

*v* = (10 − 0.4*t*) ln(*t* + 1) 0 ≤ *t* ≤ *T*

where *t* seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(*a*)find the value of *T*

**(1)**

(*b*)show that the maximum speed of the car occurs when



**(4)**

Using the iteration formula



with *t*1 = 7

(*c*)(i) find the value of *t*3 to 3 decimal places,

 (ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

**(3)**

**(Total for Question 8 is 8 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**9.**



Figure 3 shows a sketch of a parallelogram *PQRS*.

Given that

•  = 2**i** + 3**j** − 4**k**

• = 5**i** − 2**k**

(*a*)show that parallelogram *PQRS* is a rhombus.

**(2)**

(*b*)Find the exact area of the rhombus *PQRS*.

**(4)**

**(Total for Question 9 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**10.** A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, *Nb*, is modelled by the equation

*Nb* = 45 + 220 e0.05*t*

where *t* is the number of years from the start of the study.

According to the model,

(*a*)find the number of bees at the start of the study,

**(1)**

(*b*)show that, exactly 10 years after the start of the study, the number of bees was

increasing at a **rate** of approximately 18 thousand per year.

**(3)**

The number of wasps, measured in thousands, *Nw*, is modelled by the equation

*Nw* = 10 + 800 e−0.05*t*

where *t* is the number of years from the start of the study.

When *t* = *T*, according to the models, there are an equal number of bees and wasps.

(*c*)Find the value of *T* to 2 decimal places.

**(4)**

**(Total for Question 10 is 8 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**11.**



Figure 4 shows a sketch of part of the curve *C*1 with equation

*y* = 2*x*3 + 10 *x* > 0

and part of the curve *C*2 with equation

*y* = 42*x* − 15*x*2 − 7 *x* > 0

(*a*)Verify that the curves intersect at *x* = 

**(2)**

The curves intersect again at the point *P*

(*b*)Using algebra and showing all stages of working, find the exact *x* coordinate of *P*

**(5)**

**(Total for Question 11 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**12. In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Show that



where *a* and *b* are rational constants to be found.

**(5)**

**(Total for Question 12 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**13.** (i) In an arithmetic series, the first term is *a* and the common difference is *d*.

Show that

[2*a* + (*n* − 1)*d*]

**(3)**

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the

weekly amounts he saves form an arithmetic sequence.

Given that James takes *n* weeks to save exactly £64

(*a*)show that

*n*2 − 26*n* + 160 = 0

**(2)**

(*b*)Solve the equation

*n*2 − 26*n* + 160 = 0

**(1)**

(*c*)Hence state the number of weeks James takes to save enough money to buy the

printer, giving a brief reason for your answer.

**(1)**

**(Total for Question 13 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**14. In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(*a*)Given that

2 sin (*x* − 60°) = cos (*x* − 30°)

show that

tan *x* = 3

**(4)**

(*b*)Hence or otherwise solve, for 0 ≤ *θ* < 180°

2 sin 2*θ* = cos (2*θ* + 30°)

giving your answers to one decimal place.

**(4)**

**(Total for Question 14 is 8 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**15.**

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A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

• face *ABC* is a sector of a circle with radius *r* cm and centre *A*

• angle *BAC* = 0.8 radians

• faces *ABC* and *DEF* are congruent

• edges *AD*, *CF* and *BE* are perpendicular to faces *ABC* and *DEF*

• edges *AD*, *CF* and *BE* have length *h* cm

Given that the volume of the toy is 240 cm3

(*a*)show that the surface area of the toy, *S* cm2, is given by

*S* = 0.8*r*2 + 

making your method clear.

**(4)**

Using algebraic differentiation,

(*b*)find the value of *r* for which *S* has a stationary point.

**(4)**

(*c*)Prove, by further differentiation, that this value of *r* gives the minimum surface area

of the toy.

**(2)**

**(Total for Question 15 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**16.**



Figure 6 shows a sketch of the curve *C* with parametric equations

*x* = 8 sin2 *t y* = 2 sin 2*t* + 3 sin *t* 0 ≤ *t* ≤ 

The region *R*, shown shaded in Figure 6, is bounded by *C*, the *x*-axis and the line with

equation *x* = 4

(*a*)Show that the area of *R* is given by



where *a* is a constant to be found.

**(5)**

(*b*)Hence, using algebraic integration, find the exact area of *R*.

**(4)**

**(Total for Question 16 is 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**TOTAL FOR PAPER IS 100 MARKS**