**1** A curve is given by the parametric equations

$$x = 2 + t, \quad y = t^2 - 1.$$

**a** Write down expressions for $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

**b** Hence, show that $\frac{dy}{dx} = 2t$.

**2** Find and simplify an expression for $\frac{dy}{dx}$ in terms of the parameter $t$ in each case.

**a** $x = t^2, \quad y = 3t$

**b** $x = t^2 - 1, \quad y = 2t^3 + t^2$

**c** $x = 2 \sin t, \quad y = 6 \cos t$

**d** $x = 3t - 1, \quad y = 2 - \frac{1}{t}$

**e** $x = \cos 2t, \quad y = \sin t$

**f** $x = e^{t+1}, \quad y = e^{2t-1}$

**g** $x = \sin^2 t, \quad y = \cos^3 t$

**h** $x = 3 \sec t, \quad y = 5 \tan t$

**i** $x = \frac{1}{t+1}, \quad y = \frac{t}{t-1}$

**3** Find, in the form $y = mx + c$, an equation for the tangent to the given curve at the point with the given value of the parameter $t$.

**a** $x = t^3, \quad y = 3t^2, \qquad t = 1$

**b** $x = 1 - t^2, \quad y = 2t - t^2, \qquad t = 2$

**c** $x = 2 \sin t, \quad y = 1 - 4 \cos t, \qquad t = \frac{\pi}{3}$

**d** $x = \ln (4 - t), \quad y = t^2 - 5, \qquad t = 3$

**4** Show that the normal to the curve with parametric equations

$$x = \sec \theta, \quad y = 2 \tan \theta, \quad 0 \le \theta < \tfrac{\pi}{2},$$

at the point where $\theta = \frac{\pi}{3}$, has the equation

$$\sqrt{3}\,x + 4y = 10\sqrt{3}.$$

**5** A curve is given by the parametric equations

$$x = \frac{1}{t}, \quad y = \frac{1}{t+2}.$$

**a** Show that $\frac{dy}{dx} = \left(\frac{t}{t+2}\right)^2$.

**b** Find an equation for the normal to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**6** A curve has parametric equations

$$x = \sin 2t, \quad y = \sin^2 t, \quad 0 \le t < \pi.$$

**a** Show that $\frac{dy}{dx} = \frac{1}{2} \tan 2t$.

**b** Find an equation for the tangent to the curve at the point where $t = \frac{\pi}{6}$.

**7** A curve has parametric equations

$$x = 3 \cos \theta, \quad y = 4 \sin \theta, \quad 0 \le \theta < 2\pi.$$

**a** Show that the tangent to the curve at the point $(3 \cos \alpha, 4 \sin \alpha)$ has the equation

$$3y \sin \alpha + 4x \cos \alpha = 12.$$

**b** Hence find an equation for the tangent to the curve at the point $(-\frac{3}{2}, 2\sqrt{3})$.

**8** A curve is given by the parametric equations

$$x = t^2, \quad y = t(t - 2), \quad t \geq 0.$$

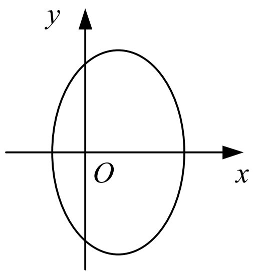
**a** Find the coordinates of any points where the curve meets the coordinate axes.

**b** Find $\frac{dy}{dx}$ in terms of $x$

**i** by first finding $\frac{dy}{dx}$ in terms of $t$,

**ii** by first finding a cartesian equation for the curve.

**9**




The diagram shows the ellipse with parametric equations

$$x = 1 - 2\cos\theta, \quad y = 3\sin\theta, \quad 0 \leq \theta < 2\pi.$$

**a** Find $\frac{dy}{dx}$ in terms of $\theta$.

**b** Find the coordinates of the points where the tangent to the curve is

**i** parallel to the $x$-axis,

**ii** parallel to the $y$-axis.

**10** A curve is given by the parametric equations

$$x = \sin\theta, \quad y = \sin 2\theta, \quad 0 \leq \theta \leq \tfrac{\pi}{2}.$$

**a** Find the coordinates of any points where the curve meets the coordinate axes.

**b** Find an equation for the tangent to the curve that is parallel to the $x$-axis.

**c** Find a cartesian equation for the curve in the form $y = f(x)$.

**11** A curve has parametric equations

$$x = \sin^2 t, \quad y = \tan t, \quad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

**a** Show that the tangent to the curve at the point where $t = \frac{\pi}{4}$ passes through the origin.

**b** Find a cartesian equation for the curve in the form $y^2 = f(x)$.

**12** A curve is given by the parametric equations

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}, \quad t \neq 0.$$

**a** Find an equation for the tangent to the curve at the point $P$ where $t = 3$.

**b** Show that the tangent to the curve at $P$ does not meet the curve again.

**c** Show that the cartesian equation of the curve can be written in the form

$$x^2 - y^2 = k,$$

where $k$ is a constant to be found.