

1 $f: x \rightarrow 3x - 5, x \in \mathbb{R}$ $g: x \rightarrow \frac{4}{6-x}, x \in \mathbb{R}, x \neq 6$ $h: x \rightarrow x^2 + 4x - 1, x \in \mathbb{R}$

Find the value of

a $f(3)$ **b** $g(4)$ **c** $h(2)$ **d** $f(1)$ **e** $h(-1)$ **f** $g(8)$
g $g(-4)$ **h** $f(\frac{2}{3})$ **i** $h(\frac{1}{2})$ **j** $f(-1)$ **k** $h(-3)$ **l** $g(1\frac{2}{3})$

2 $f: x \rightarrow \ln(2 - 5x), x \in \mathbb{R}, x < 0.4$ $g: x \rightarrow \sin(2x + \frac{\pi}{3}), x \in \mathbb{R}$ $h: x \rightarrow 3 + 2e^{1-x}, x \in \mathbb{R}$

Find, correct to 3 significant figures where appropriate, the value of

a $g(\frac{\pi}{3})$ **b** $f(0)$ **c** $h(1)$ **d** $g(\frac{\pi}{6})$ **e** $h(2)$ **f** $f(-\frac{1}{2})$
g $h(-0.8)$ **h** $f(0.2)$ **i** $g(0.3)$ **j** $h(\frac{2}{3})$ **k** $g(-1)$ **l** $f(-\frac{3}{4})$

3 Sketch each function and state its range.

a $f: x \rightarrow 2x + 1, x \in \mathbb{R}, 0 \leq x \leq 7$ **b** $f: x \rightarrow 3x - 2, x \in \mathbb{R}, x \geq 0$
c $f: x \rightarrow 5 - x, x \in \mathbb{R}, -5 \leq x \leq 5$ **d** $f: x \rightarrow 4 - 7x, x \in \mathbb{R}$
e $f: x \rightarrow x^2, x \in \mathbb{R}, -3 < x < 3$ **f** $f: x \rightarrow x^2 + 3, x \in \mathbb{R}$
g $f: x \rightarrow x^2 - 6, x \in \mathbb{R}, x \geq 0$ **h** $f: x \rightarrow (x - 1)^2, x \in \mathbb{R}, -2 \leq x \leq 4$
i $f: x \rightarrow (x + 2)^2, x \in \mathbb{R}$ **j** $f: x \rightarrow 4 - x^2, x \in \mathbb{R}$
k $f: x \rightarrow x^3, x \in \mathbb{R}, -10 < x \leq 10$ **l** $f: x \rightarrow -x^3, x \in \mathbb{R}$

4 Sketch each function and state its range.

a $f: x \rightarrow x^2 + 2x - 8, x \in \mathbb{R}$ **b** $f: x \rightarrow \frac{1}{x}, x \in \mathbb{R}, x \neq 0$
c $f: x \rightarrow \frac{1}{x^2}, x \in \mathbb{R}, x \neq 0$ **d** $f: x \rightarrow \cos x, x \in \mathbb{R}, 0 \leq x \leq 2\pi$
e $f: x \rightarrow 5^x, x \in \mathbb{R}$ **f** $f: x \rightarrow \tan x, x \in \mathbb{R}, -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

5 Find the domain of each function given its range.

a $f: x \rightarrow x - 1, f(x) \in \mathbb{R}, -1 \leq f(x) < 6$ **b** $f: x \rightarrow 4 - 3x, f(x) \in \mathbb{R}, f(x) \leq 4$
c $f: x \rightarrow x^3, f(x) \in \mathbb{R}, 0 \leq f(x) \leq 125$ **d** $f: x \rightarrow \frac{1}{x}, f(x) \in \mathbb{R}, 2 < f(x) < 10$

6 Given that for $x \in \mathbb{R}$, $f(x) \equiv 4x + 3$, $g(x) \equiv x^2 - 7$ and $h(x) \equiv \frac{9}{x+2}, x \neq -2$, solve the equations

a $f(x) = 9$ **b** $g(x) = 18$ **c** $h(x) = 6$
d $f(x) = h(x)$ **e** $g(x) - \frac{1}{h(x)} = -6\frac{1}{3}$ **f** $f(x) + g(x) = 0$

7 Express each function in the form indicated and hence, state its range.

a $f: x \rightarrow x^2 + 4x + 11, x \in \mathbb{R}$ in the form $(x + a)^2 + b$
b $f: x \rightarrow x^2 - 2x - 6, x \in \mathbb{R}$ in the form $(x + a)^2 + b$
c $f: x \rightarrow 4x^2 + 12x + 3, x \in \mathbb{R}$ in the form $(ax + b)^2 + c$
d $f: x \rightarrow 9x^2 - 6x + 16, x \in \mathbb{R}$ in the form $(ax + b)^2 + c$
e $f: x \rightarrow 15 - 4x - x^2, x \in \mathbb{R}$ in the form $a - (x + b)^2$

1 $f: x \rightarrow 4x - 3, x \in \mathbb{R}$ $g: x \rightarrow 2 - x, x \in \mathbb{R}$ $h: x \rightarrow x^2 + 5, x \in \mathbb{R}$

Evaluate

a $gf(2)$ **b** $gh(1)$ **c** $fg(-3)$ **d** $hf(3)$
e $gg(5)$ **f** $ff(\frac{1}{2})$ **g** $hg(8)$ **h** $fh(1\frac{1}{2})$

2 $f: x \rightarrow 5x + 2, x \in \mathbb{R}$ $g: x \rightarrow \cos x, x \in \mathbb{R}$ $h: x \rightarrow \ln x, x \in \mathbb{R}, x > 0$

Evaluate, giving your answers to 3 significant figures

a $fh(20)$ **b** $gh(3)$ **c** $fg(5)$ **d** $gg(-4)$
e $gf(1\frac{3}{4})$ **f** $hg(6.7)$ **g** $hh(50)$ **h** $hf(-0.3)$

3 $f: x \rightarrow 2x + 1, x \in \mathbb{R}$ $g: x \rightarrow 1 - 3x, x \in \mathbb{R}$ $h: x \rightarrow x^2 + 4, x \in \mathbb{R}$

Given the functions f , g and h , express the following composite functions in a similar form.

a fg **b** ff **c** fh **d** hf
e gh **f** gg **g** hg **h** gf

4 $f: x \rightarrow 4 - x, x \in \mathbb{R}$ $g: x \rightarrow e^x, x \in \mathbb{R}$ $h: x \rightarrow 2x^2 + 7, x \in \mathbb{R}$

Given the functions f , g and h , express the following composite functions in a similar form.

a gf **b** hg **c** fh **d** gg
e gh **f** ff **g** fg **h** hf

5 $f: x \rightarrow 5x - 3, x \in \mathbb{R}$ $g: x \rightarrow 3x^2 + 1, x \in \mathbb{R}$ $h: x \rightarrow \frac{1}{x-2}, x \in \mathbb{R}, x \neq 2$

Solve

a $ff(x) = -8$ **b** $hf(x) = 2$ **c** $gf(x) = 28$ **d** $hg(x) = \frac{1}{2}$
e $fh(x) = 7$ **f** $fg(x) = 32$ **g** $gh(x) = 4$ **h** $hh(x) = -2$

6 $f: x \rightarrow \ln x, x \in \mathbb{R}, x > 0$ $g: x \rightarrow 3 + 2x, x \in \mathbb{R}$ $h: x \rightarrow e^x, x \in \mathbb{R}$

Solve, giving your answers to 2 decimal places,

a $gh(x) = 9$ **b** $fg(x) = 3.6$ **c** $hg(x) = 4$ **d** $gf(x) = 10.4$

7 The functions f and g are defined by

$$f: x \rightarrow \frac{x+1}{5}, x \in \mathbb{R} \qquad g: x \rightarrow e^x, x \in \mathbb{R}$$

- a** State the range of g .
b Solve $fg(x) = 17$.

8 The functions f and g are defined by

$$f(x) \equiv 4x - 9, x \in \mathbb{R} \qquad g(x) \equiv x^2, x \in \mathbb{R}$$

- a** Evaluate $ff(3\frac{1}{4})$.
b Solve $gf(x) = 25$.
c Sketch the graph of $y = fg(x)$, showing the coordinates of any points of intersection with the coordinate axes.

9 $f: x \rightarrow \tan x, x \in \mathbb{R}$ $g: x \rightarrow 4 + \ln x, x \in \mathbb{R}^+$ $h: x \rightarrow e^{2x-1}, x \in \mathbb{R}$

Evaluate

a $gf(\frac{\pi}{4})$ **b** $hg(e^{-2})$ **c** $gh(-1)$ **d** $ff(1)$
e $hf(0.2)$ **f** $fg(7)$ **g** $hh(\frac{1}{4})$ **h** $fg(e^e)$

10 $f: x \rightarrow 3e^x + 2, x \in \mathbb{R}$ $g: x \rightarrow 4x + 1, x \in \mathbb{R}$ $h: x \rightarrow \frac{1}{x+1}, x \in \mathbb{R}, x \neq -1$

Express the following composite functions in a similar form, stating the domain in each case.

a fg **b** gf **c** hf **d** gg
e hg **f** gh **g** hh **h** ggg

11 $f: x \rightarrow \sqrt{x+4}, x \in \mathbb{R}, x > -4$ $g: x \rightarrow e^{1+2x}, x \in \mathbb{R}$ $h: x \rightarrow \frac{x+1}{3}, x \in \mathbb{R}$

Solve

a $fh(x) = 3$ **b** $fg(x) = 7$ **c** $gh(x) = 11$ **d** $hh(x) = \frac{2}{3}$
e $hg(x) = 1.2$ **f** $hf(x) = \frac{1}{2}$ **g** $ff(x) = 3$ **h** $ghh(x) = \frac{1}{2}$

12 $f(x) \equiv x^3, x \in \mathbb{R}$ $g(x) \equiv x + 2, x \in \mathbb{R}$

Find the composition of the functions f and g that corresponds to the function h , where

a $h(x) \equiv (x+2)^3, x \in \mathbb{R}$ **b** $h(x) \equiv x^3 + 2, x \in \mathbb{R}$ **c** $h(x) \equiv x + 4, x \in \mathbb{R}$
d $h(x) \equiv x^9, x \in \mathbb{R}$ **e** $h(x) \equiv x^9 + 2, x \in \mathbb{R}$ **f** $h(x) \equiv (x+2)^3 + 2, x \in \mathbb{R}$

13 $f(x) \equiv x - 4, x \in \mathbb{R}$ $g(x) \equiv 3x^2, x \in \mathbb{R}$ $h(x) \equiv \frac{1}{x}, x \in \mathbb{R}, x \neq 0$

Find the composition of the functions f, g and h that corresponds to the function j , where

a $j(x) \equiv 3x^2 - 4, x \in \mathbb{R}$ **b** $j(x) \equiv \frac{1}{x-4}, x \in \mathbb{R}, x \neq 4$
c $j(x) \equiv \frac{3}{x^2}, x \in \mathbb{R}, x \neq 0$ **d** $j(x) \equiv 27x^4, x \in \mathbb{R}$
e $j(x) \equiv \frac{1}{3x^2} - 4, x \in \mathbb{R}, x \neq 0$ **f** $j(x) \equiv \frac{1}{3x^2 - 4}, x \in \mathbb{R}, x \neq \pm \frac{2}{\sqrt{3}}$

14 The functions f and g are defined by

$f: x \rightarrow 5^x - 7, x \in \mathbb{R}$ $g: x \rightarrow 2x + 3, x \in \mathbb{R}$

- a** Find and simplify an expression for gf , stating its domain.
b Solve the equation $gf(x) = 10$.

15 The functions f and g are defined by

$f: x \rightarrow 2(x+1), x \in \mathbb{R}$ $g: x \rightarrow x^2 - 9, x \in \mathbb{R}$

- a** Express gf in terms of x and state its domain and range.
b Sketch the graph of $y = gf(x)$, showing the coordinates of any points of intersection with the coordinate axes.

The equation $gf(x) - 2f(x) = a$, where a is a constant, has no real roots.

- c** Show that $a < -10$.

1 The domain of each of the following functions is $x \in \mathbb{R}$. For each function, find its inverse $f^{-1}(x)$.

a $f: x \rightarrow 10x + 3$ **b** $f: x \rightarrow 9 + 2x$ **c** $f: x \rightarrow 5 - 6x$
d $f: x \rightarrow \frac{x+3}{4}$ **e** $f: x \rightarrow \frac{1}{3}(2x - 5)$ **f** $f: x \rightarrow 8 - \frac{3}{5}x$

2 For each function, find $f^{-1}(x)$ and state its domain.

a $f(x) \equiv \ln x, x \in \mathbb{R}, x > 0$ **b** $f(x) \equiv \frac{1}{x}, x \in \mathbb{R}, x \neq 0$
c $f(x) \equiv \sqrt[4]{x}, x \in \mathbb{R}, x > 0$ **d** $f(x) \equiv 3x - 4, x \in \mathbb{R}, 0 \leq x < 3$
e $f(x) \equiv \frac{1}{x-5}, x \in \mathbb{R}, x \neq 5$ **f** $f(x) \equiv 2 + \frac{1}{x}, x \in \mathbb{R}, x \neq 0$

3 For each of the following functions,

- i** find, in the form $f^{-1}: x \rightarrow \dots$, the inverse function of f and state its domain,
ii sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

a $f: x \rightarrow 2x + 1, x \in \mathbb{R}$ **b** $f: x \rightarrow \frac{1-x}{5}, x \in \mathbb{R}$ **c** $f: x \rightarrow \frac{10}{x}, x \in \mathbb{R}, x \neq 0$
d $f: x \rightarrow x^2, x \in \mathbb{R}, x > 0$ **e** $f: x \rightarrow e^x, x \in \mathbb{R}$ **f** $f: x \rightarrow x^3, x \in \mathbb{R}$

4 For each of the following, solve the equation $f^{-1}(x) = g(x)$.

a $f: x \rightarrow 5x + 1, x \in \mathbb{R}$ $g: x \rightarrow 2, x \in \mathbb{R}$
b $f: x \rightarrow \frac{2x-4}{3}, x \in \mathbb{R}$ $g: x \rightarrow 7 - x, x \in \mathbb{R}$
c $f: x \rightarrow e^x + 2, x \in \mathbb{R}$ $g: x \rightarrow \ln(3x - 8), x \in \mathbb{R}, x > \frac{8}{3}$
d $f: x \rightarrow \sqrt{x+2}, x \in \mathbb{R}, x \geq -2$ $g: x \rightarrow 3x - 4, x \in \mathbb{R}$
e $f: x \rightarrow \frac{4}{x+3}, x \in \mathbb{R}, x \neq -3$ $g: x \rightarrow 5(x+1), x \in \mathbb{R}$

5 The function f is defined by $f: x \rightarrow 4 - 2x, x \in \mathbb{R}$.

- a** Sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.
b Find the coordinates of the point where the lines $y = f(x)$ and $y = f^{-1}(x)$ intersect.

6 The functions f and g are defined by

$$f: x \rightarrow 3 - 2x, x \in \mathbb{R} \qquad g: x \rightarrow \frac{1}{2x+4}, x \in \mathbb{R}, x \neq -2$$

- a** Find $g^{-1}(x)$ and state its domain and range.
b Express gf in terms of x and state its domain.
c Solve the equation $gf(x) = f^{-1}(x)$.

7 The functions f and g are defined by

$$f: x \rightarrow 5x + 2, x \in \mathbb{R} \qquad g: x \rightarrow \frac{1}{x}, x \in \mathbb{R}, x \neq 0$$

- a** Find the following functions, stating the domain in each case.
i f^{-1} **ii** fg **iii** $(fg)^{-1}$
b Solve the equation $f^{-1}(x) = fg(x)$, giving your answers correct to 2 decimal places.

8 For each of the following functions, find the inverse function in the form $f^{-1}: x \rightarrow \dots$ and state its domain.

a $f: x \rightarrow \frac{1}{2} \ln(4x - 9), x \in \mathbb{R}, x > 2\frac{1}{4}$

b $f: x \rightarrow \frac{x-2}{x+5}, x \in \mathbb{R}, x \neq -5$

c $f: x \rightarrow e^{0.4x-2}, x \in \mathbb{R}$

d $f: x \rightarrow \sqrt[3]{x^5 - 3}, x \in \mathbb{R}$

e $f: x \rightarrow \log_{10}(2 - 7x), x \in \mathbb{R}, x < \frac{2}{7}$

f $f: x \rightarrow \frac{4-x}{3x+2}, x \in \mathbb{R}, x \neq -\frac{2}{3}$

9 For each of the following functions,

i find, in the form $f^{-1}: x \rightarrow \dots$, the inverse function of f and state its domain,

ii sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

a $f: x \rightarrow e^{2x}, x \in \mathbb{R}$

b $f: x \rightarrow x^2 + 4, x \in \mathbb{R}, x > 0$

c $f: x \rightarrow \ln(x - 3), x \in \mathbb{R}, x > 3$

d $f: x \rightarrow x^2 + 6x + 9, x \in \mathbb{R}, x > -3$

10 For each of the following functions,

i find the range of f ,

ii find $f^{-1}(x)$, stating its domain.

a $f(x) \equiv x^2 + 6x + 3, x \in \mathbb{R}, x < -3$

b $f(x) \equiv x^2 - 4x + 5, x \in \mathbb{R}, x \geq 2$

c $f(x) \equiv x^2 + 5x - 2, x \in \mathbb{R}, x < -2\frac{1}{2}$

d $f(x) \equiv x^2 - 3x + 5, x \in \mathbb{R}, 2 < x < 4$

e $f(x) \equiv (2 - x)(4 + x), x \in \mathbb{R}, x \geq -1$

f $f(x) \equiv 20x - 5x^2, x \in \mathbb{R}, x > 2$

11 For each of the following, solve the equation $f^{-1}(x) = g(x)$.

a $f: x \rightarrow \frac{1}{3}(2x - 5), x \in \mathbb{R}$

g $g: x \rightarrow \frac{4}{2-x}, x \in \mathbb{R}, x \neq 2$

b $f: x \rightarrow \ln \frac{x+3}{5}, x \in \mathbb{R}, x > -3$

g $g: x \rightarrow 10 - 6e^{-x}, x \in \mathbb{R}$

c $f: x \rightarrow x^2 - 4, x \in \mathbb{R}, x > 0$

g $g: x \rightarrow \frac{x+6}{3}, x \in \mathbb{R}$

12 The function f is defined by

$$f: x \rightarrow \frac{x+b}{x+a}, x \in \mathbb{R}, x \neq 2.$$

a State the value of the constant a .

Given that $f(6) = 4$,

b find the value of the constant b ,

c find $f^{-1}(x)$ and state its domain.

13 The functions f and g are defined by

$$f: x \rightarrow x^2 - 3x, x \in \mathbb{R}, x \geq 1\frac{1}{2},$$

$$g: x \rightarrow 2x + 3, x \in \mathbb{R}.$$

a Find, in the form $f^{-1}: x \rightarrow \dots$, the inverse function of f and state its domain.

b On the same set of axes, sketch $y = f(x)$ and $y = f^{-1}(x)$.

Given that $f^{-1}g^{-1}(12) = a(1 + \sqrt{3})$,

c show that $a = 1\frac{1}{2}$.