C3 DIFFERENTIATION — Worksheet A

1 Differentiate with respect to x

a e^x **b** $3e^x$ **c** $\ln x$ **d** $\frac{1}{2}\ln x$

2 Differentiate with respect to t

a $7 - 2e^t$ **b** $3t^2 + \ln t$ **c** $e^t + t^5$ **d** $t^{\frac{3}{2}} + 2e^t$

e $2\ln t + \sqrt{t}$ **f** $2.5e^t - 3.5\ln t$ **g** $\frac{1}{t} + 8\ln t$ **h** $7t^2 - 2t + 4e^t$

3 Find $\frac{d^2y}{dx^2}$ for each of the following.

a $y = 4x^3 + e^x$ **b** $y = 7e^x - 5x^2 + 3x$ **c** $y = \ln x + x^{\frac{5}{2}}$

d $y = 5e^x + 6\ln x$ **e** $y = \frac{3}{x} + 3\ln x$ **f** $y = 4\sqrt{x} + \frac{1}{4}\ln x$

4 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = 3x + e^x$, $x = 0$ **b** $f(x) = \ln x - x^2$, $x = 4$

c $f(x) = x^{\frac{1}{2}} + 2\ln x$, $x = 9$ **d** $f(x) = 5e^x + \frac{1}{x^2}$, $x = -\frac{1}{2}$

5 Find, in each case, any values of x for which $\frac{dy}{dx} = 0$.

a $y = 5\ln x - 8x$ **b** $y = 2.4e^x - 3.6x$ **c** $y = 3x^2 - 14x + 4\ln x$

6 Find the value of x for which $f'(x)$ takes the value indicated in each case.

a $f(x) = 2e^x - 3x$, $f'(x) = 7$ **b** $f(x) = 15x + \ln x$, $f'(x) = 23$

c $f(x) = \frac{x^2}{8} - 2x + \ln x$, $f'(x) = -1$ **d** $f(x) = 30\ln x - x^2$, $f'(x) = 4$

7 Find the coordinates and the nature of any stationary points on each of the following curves.

a $y = e^x - 2x$ **b** $y = \ln x - 10x$ **c** $y = 2\ln x - \sqrt{x}$

d $y = 4x - 5e^x$ **e** $y = 7 + 2x - 4\ln x$ **f** $y = x^2 - 26x + 72\ln x$

8 Given that $y = x + ke^x$, where k is a constant, show that

$$(1 - x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

9 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

a $y = e^x$, $x = 2$ **b** $y = \ln x$, $x = 3$ **c** $y = 0.8x - 2e^x$, $x = 0$

d $y = 5\ln x + \frac{4}{x}$, $x = 1$ **e** $y = x^{\frac{1}{3}} - 3e^x$, $x = 1$ **f** $y = \ln x - \sqrt{x}$, $x = 9$

10 Find an equation for the normal to each curve at the point on the curve with the given x-coordinate.

a $y = \ln x$, $x = e$ **b** $y = 4 + 3e^x$, $x = 0$ **c** $y = 10 + \ln x$, $x = 3$

d $y = 3\ln x - 2x$, $x = 1$ **e** $y = x^2 + 8\ln x$, $x = 1$ **f** $y = \frac{1}{10}x - \frac{3}{10}e^x - 1$, $x = 0$

1 **a** Find an equation for the normal to the curve $y = \frac{2}{5}x + \frac{1}{10}e^x$ at the point on the curve where $x = 0$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Find the coordinates of the point where this normal crosses the x-axis.

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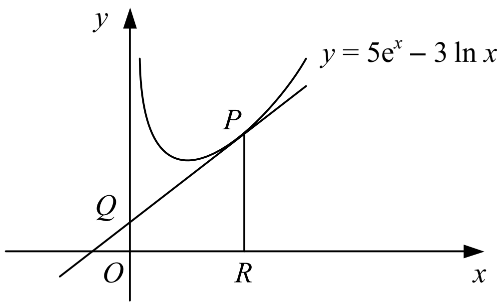


The diagram shows the curve with equation $y = 5e^x - 3\ln x$ and the tangent to the curve at the point P with x-coordinate 1.

a Show that the tangent at P has equation $y = (5e - 3)x + 3$.

The tangent at P meets the y-axis at Q.
The line through P parallel to the y-axis meets the x-axis at R.

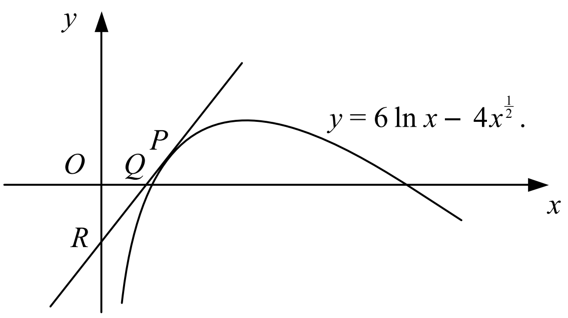
b Find the area of trapezium $ORPQ$, giving your answer in terms of e.

3 A curve has equation $y = 3x - \frac{1}{2}e^x$.

a Find the coordinates of the stationary point on the curve, giving your answers in terms of natural logarithms.

b Determine the nature of the stationary point.

4



The diagram shows the curve $y = 6\ln x - 4x^{\frac{1}{2}}$. The x-coordinate of the point P on the curve is 4.

The tangent to the curve at P meets the x-axis at Q and the y-axis at R.

a Find an equation for the tangent to the curve at P.

b Hence, show that the area of triangle OQR is $(10 - 12\ln 2)^2$.

5 The curve with equation $y = 2x - 2 - \ln x$ passes through the point A (1, 0). The tangent to the curve at A crosses the y-axis at B and the normal to the curve at A crosses the y-axis at C.

a Find an equation for the tangent to the curve at A.

b Show that the mid-point of BC is the origin.

The curve has a minimum point at D.

c Show that the y-coordinate of D is $\ln 2 - 1$.

6 **a** Sketch the curve with equation $y = e^x + k$, where k is a positive constant.

Show on your sketch the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes.

b Find an equation for the tangent to the curve at the point on the curve where $x = 2$.

Given that the tangent passes through the x-axis at the point $(-1, 0)$,

c show that $k = 2e^2$.

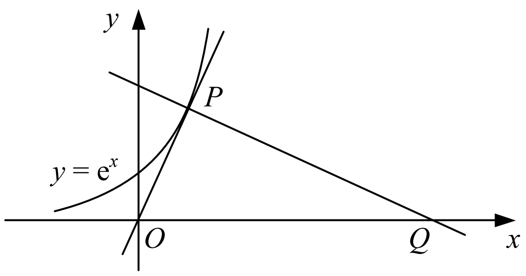
7 A curve has equation $y = 3x^2 - 2 \ln x$, $x > 0$.

The gradient of the curve at the point P on the curve is -1.

a Find the x-coordinate of P.

b Find an equation for the tangent to the curve at the point on the curve where $x = 1$.

8



The diagram shows the curve with equation $y = e^x$ which passes through the point $P\ (p, e^p)$. Given that the tangent to the curve at P passes through the origin and that the normal to the curve at P meets the x-axis at Q,

a show that $p = 1$,

b show that the area of triangle OPQ, where O is the origin, is $\frac{1}{2}e(1 + e^2)$.

9 The curve with equation $y = 4 - e^x$ meets the y-axis at the point P and the x-axis at the point Q.

a Find an equation for the normal to the curve at P.

b Find an equation for the tangent to the curve at Q.

The normal to the curve at P meets the tangent to the curve at Q at the point R.

The x-coordinate of R is $a \ln 2 + b$, where a and b are rational constants.

c Show that $a = \frac{8}{5}$.

d Find the value of b.

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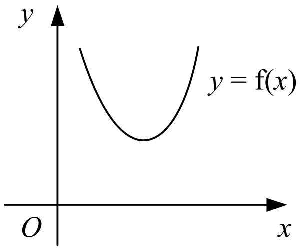

The diagram shows a sketch of the curve $y = f(x)$ where

$$f : x \rightarrow 9x^4 - 16 \ln x, \quad x > 0.$$

Given that the set of values of x for which $f(x)$ is a decreasing function of x is $0 < x \leq k$, find the exact value of k.