

- 1 a 20.1 b 0.135 c 13.6 d -0.598 e 1.97 f 0.434
- 2 a = 4 b = $e^{\ln 3} = 3$ c = $2e^{\ln \frac{1}{2}} = \frac{1}{3}$ d = 7 e = $\ln e^{-1} = -1$ f = -0.5
- 3 a $x = 4$ b $x = 17$ c $x^2 = 25$
 $x > 0 \therefore x = 5$ d $\frac{1}{x} = \frac{1}{3}$
 $x = 3$
- 4 a $x = e^{15}$ b $\ln t = 6$
 $t = e^6$ c $x - 4 = e^7$
 $x = e^7 + 4$
- d $\ln 5y = 8$ e $\frac{1}{2}x + 3 = e^{2.5}$ f $4 - 3x = e^{11}$
 $5y = e^8$ $\frac{1}{2}x = e^{2.5} - 3$ $3x = 4 - e^{11}$
 $y = \frac{1}{5}e^8$ $x = 2e^{2.5} - 6$ $x = \frac{1}{3}(4 - e^{11})$
- 5 a $x = \ln 0.7$ b $e^y = 2$
 $y = \ln 2$ c $5x = \ln 3$
 $x = \frac{1}{5} \ln 3$
- d $4t + 1 = \ln 12$ e $e^{2x-3} = 14$ f $e^{4-5x} = \frac{7}{2}$
 $t = \frac{1}{4}(\ln 12 - 1)$ $2x - 3 = \ln 14$ $4 - 5x = \ln \frac{7}{2}$
 $x = \frac{1}{2}(\ln 14 + 3)$ $x = \frac{1}{5}(4 - \ln \frac{7}{2})$
- 6 a $e^x = 12$ b $15x - 7 = e^4$ c $e^{\frac{1}{2}y+3} = \frac{11}{4}$
 $x = \ln 12 = 2.48$ $x = \frac{1}{15}(e^4 + 7) = 4.11$ $\frac{1}{2}y + 3 = \ln \frac{11}{4}$
 $y = 2(\ln \frac{11}{4} - 3) = -3.98$
- d $\ln(5 - 2x) = \frac{7}{3}$ e $10 - 3y = e^e$ f $2 \ln x + 3 \ln x = 19$
 $5 - 2x = e^{\frac{7}{3}}$ $y = \frac{1}{3}(10 - e^e) = -1.72$ $\ln x = \frac{19}{5}$
 $x = \frac{1}{2}(5 - e^{\frac{7}{3}}) = -2.66$ $x = e^{\frac{19}{5}} = 44.70$
- g $e^{\frac{2}{3}x} = 3$ h $e^{3t-1} = 4$ i $\ln \frac{2x-5}{x} = \frac{1}{4}$
 $\frac{2}{3}x = \ln 3$ $3t - 1 = \ln 4$ $2x - 5 = e^{\frac{1}{4}}x$
 $x = \frac{4}{9} \ln 3 = 0.49$ $t = \frac{1}{3}(\ln 4 + 1) = 0.80$ $(2 - e^{\frac{1}{4}})x = 5$
 $x = \frac{5}{2 - e^{\frac{1}{4}}} = 6.98$
- 7 $2e^{2x} - 11e^x + 12 = 0$
 $(2e^x - 3)(e^x - 4) = 0$
 $e^x = \frac{3}{2}, 4$
 $x = \ln \frac{3}{2}, \ln 4$

8 a $= \frac{(3x-4)(x-2)}{(x-2)(x-3)} = \frac{3x-4}{x-3}$

b $\ln \frac{3x^2-10x+8}{x^2-5x+6} = \ln 2x$

$$\frac{3x^2-10x+8}{x^2-5x+6} = 2x$$

$$\frac{3x-4}{x-3} = 2x$$

$$3x-4 = 2x(x-3)$$

$$2x^2-9x+4=0$$

$$(2x-1)(x-4)=0$$

$$x = \frac{1}{2}, 4$$

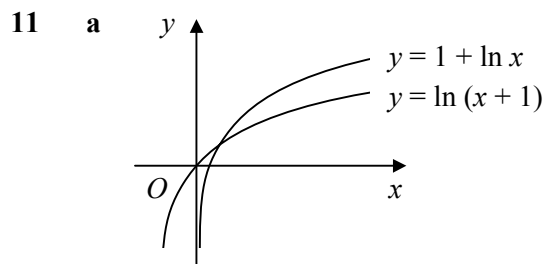
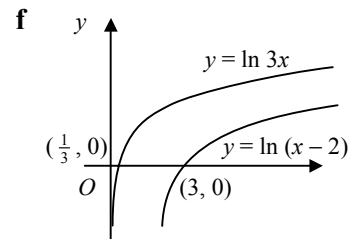
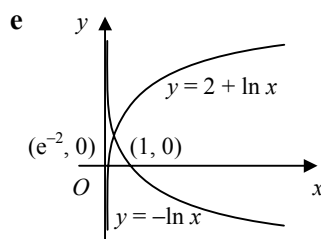
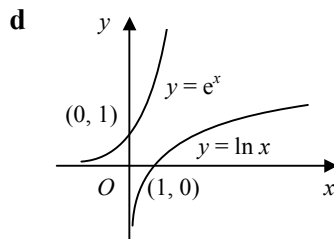
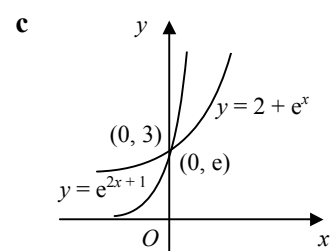
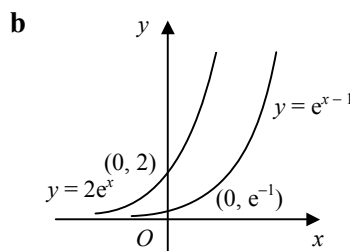
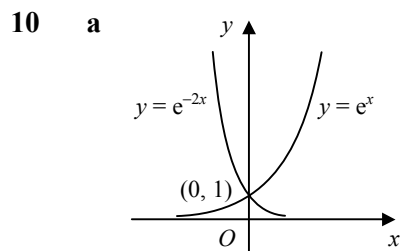
9 $e^{5y} - x = 0 \Rightarrow 5y = \ln x$

$$\ln x^4 = 7 - y \Rightarrow 4 \ln x = 7 - y$$

sub. $20y = 7 - y$

$$y = \frac{1}{3}$$

$$\therefore x = e^{\frac{5}{3}} = 5.29, y = 0.33$$



b $\ln(x+1) = 1 + \ln x$

$$\ln(x+1) - \ln x = 1$$

$$\ln \frac{x+1}{x} = 1$$

$$\frac{x+1}{x} = e$$

$$x+1 = ex$$

$$1 = x(e-1)$$

$$x = \frac{1}{e-1}$$

12 a 3

$$\text{b } x = 0 \therefore y = 3 + e^{-1}$$

$$\therefore (0, 3 + e^{-1})$$

$$\text{c } 3 + e^{2x-1} = 7$$

$$e^{2x-1} = 4$$

$$2x - 1 = \ln 4$$

$$x = \frac{1}{2}(1 + \ln 4)$$

$$x = \frac{1}{2} + \ln 2$$

13 a $t = 10, N = 50e^{-2} = 6.77$ (3sf)

$$\text{b } 3 = 50e^{-0.2t}$$

$$t = -5 \ln \frac{3}{50} = 14.1$$
 (3sf)

14 a $160 = 240e^{180k}$

$$k = \frac{1}{180} \ln \frac{2}{3} = -0.00225$$
 (3sf)

$$\text{b } m = 240e^{-0.002253t}$$

$$120 = 240e^{-0.002253t}$$

$$t = \frac{-1}{0.002253} \ln \frac{1}{2} = 308$$
 years (3sf)

15 a $t = 15, N = 20e^{0.6} = 36.4$ (3sf)

$$\text{b i } k = 20e^{0.04t}$$

$$t = \frac{\ln(\frac{k}{20})}{0.04} = 25 \ln \frac{k}{20}$$

$$\text{ii } 2k = 20e^{0.04t}$$

$$t = \frac{\ln(\frac{k}{10})}{0.04} = 25 \ln \frac{k}{10}$$

c time for N to increase from k to $2k$

$$= 25 \ln \frac{k}{10} - 25 \ln \frac{k}{20}$$

$$= 25 \ln \frac{(\frac{k}{10})}{(\frac{k}{20})}$$

$$= 25 \ln 2$$

\therefore time for N to double is constant

16 a $300 = N_0e^{10k} \Rightarrow N_0 = \frac{300}{e^{10k}}$

$$225 = N_0e^{20k}$$

$$\therefore 225 = \frac{300}{e^{10k}} \times e^{20k}$$

$$e^{10k} = \frac{3}{4}$$

$$k = \frac{1}{10} \ln \frac{3}{4} = -0.0288$$
 (3sf)

$$N_0 = \frac{300}{\frac{3}{4}} = 400$$

$$\text{b } N = 400e^{-0.02877t}$$

$$150 = 400e^{-0.02877t}$$

$$t = \frac{-1}{0.02877} \ln \frac{3}{8} = 34.1$$
 (3sf)