1

a $= 5(x + 3)^4$

b $= 3(2x - 1)^2 \times 2$
$= 6(2x - 1)^2$

c $= 7(8 - x)^6 \times (-1)$
$= -7(8 - x)^6$

d $= 12(3x + 4)^5 \times 3$
$= 36(3x + 4)^5$

e $= 4(6 - 5x)^3 \times (-5)$
$= -20(6 - 5x)^3$

f $= -(x - 2)^{-2}$

g $= -12(2x + 3)^{-4} \times 2$
$= -24(2x + 3)^{-4}$

h $= -2(7 - 3x)^{-3} \times (-3)$
h $= 6(7 - 3x)^{-3}$

2

a $= 6e^{3t}$

b $= \frac{1}{2}(4t - 1)^{-\frac{1}{2}} \times 4$
$= 2(4t - 1)^{-\frac{1}{2}}$

c $= \frac{5}{t}$

d $= \frac{3}{2}(8 - 3t)^{\frac{1}{2}} \times (-3)$
$= -\frac{9}{2}(8 - 3t)^{\frac{1}{2}}$

e $= \frac{3}{6t+1} \times 6$
$= \frac{18}{6t+1}$

f $= \frac{1}{2}e^{5t+4} \times 5$
$= \frac{5}{2}e^{5t+4}$

g $= \frac{d}{dx}[6(2t - 5)^{-\frac{1}{3}}]$
$= -2(2t - 5)^{-\frac{4}{3}} \times 2$
$= -4(2t - 5)^{-\frac{4}{3}}$

h $= \frac{2}{3-\frac{1}{4}t} \times (-\frac{1}{4})$
$= \frac{2}{t-12}$

3

a $\frac{dy}{dx} = 4(3x - 1)^3 \times 3$
$= 12(3x - 1)^3$
$\frac{d^2y}{dx^2} = 36(3x - 1)^2 \times 3$
$= 108(3x - 1)^2$

b $\frac{dy}{dx} = \frac{4}{1+2x} \times 2$
$= 8(1 + 2x)^{-1}$
$\frac{d^2y}{dx^2} = -8(1 + 2x)^{-2} \times 2$
$= \frac{-16}{(1+2x)^2}$

c $\frac{dy}{dx} = \frac{1}{2}(5-2x)^{-\frac{1}{2}} \times (-2)$
$= -(5-2x)^{-\frac{1}{2}}$
$\frac{d^2y}{dx^2} = \frac{1}{2}(5-2x)^{-\frac{3}{2}} \times (-2)$
$= -(5-2x)^{-\frac{3}{2}}$

4

a $f'(x) = 2x - \frac{6}{x}$
$f'(3) = 6 - 2 = 4$

b $f'(x) = 2 - e^{x-2}$
$f'(2) = 2 - 1 = 1$

c $f'(x) = 4(2 - 5x)^3 \times (-5) = -20(2 - 5x)^3$
$f'(\frac{1}{2}) = -20 \times (-\frac{1}{8}) = \frac{5}{2}$

d $f'(x) = -4(x + 5)^{-2}$
$f'(-1) = -4 \times \frac{1}{16} = -\frac{1}{4}$

5

a $f'(x) = 2(3x+15)^{-\frac{1}{2}} \times 3 = 2$
$\frac{6}{\sqrt{3x+15}} = 2$
$\sqrt{3x+15} = 3$
$3x + 15 = 9$
$x = -2$

b $f'(x) = 2x - \frac{1}{x-2} = 5$
$2x(x - 2) - 1 = 5(x - 2)$
$2x^2 - 9x + 9 = 0$
$(2x - 3)(x - 3) = 0$
for real $f(x)$, $x > 2$ $\therefore x = 3$

6

a $= 3(x^2 - 4)^2 \times 2x$
$= 6x(x^2 - 4)^2$

b $= 12(3x^2 + 1)^5 \times 6x$
$= 72x(3x^2 + 1)^5$

c $= \frac{1}{3+2x^2} \times 4x$
$= \frac{4x}{3+2x^2}$

d $= \frac{d}{dx}[(4 - x^2)^3]$
$= 3(4 - x^2)^2 \times (-2x)$
$= -6x(4 - x^2)^2$

e $= \frac{d}{dx}[(\frac{1}{2}x^4 + 3)^8]$
$= 8(\frac{1}{2}x^4 + 3)^7 \times 2x^3$
$= 16x^3(\frac{1}{2}x^4 + 3)^7$

f $= \frac{d}{dx}[(3 - x^2)^{-\frac{1}{2}}]$
$= -\frac{1}{2}(3-x^2)^{-\frac{3}{2}} \times (-2x)$
$= x(3 - x^2)^{-\frac{3}{2}}$

g $= 7e^{x^2} \times 2x$
$= 14xe^{x^2}$

h $= 4(1-5x+x^3)^3 \times (-5+3x^2)$
$= 4(3x^2 - 5)(1 - 5x + x^3)^3$

i $= \frac{3}{4-\sqrt{x}} \times (-\frac{1}{2}x^{-\frac{1}{2}})$
$= \frac{3}{2x - 8\sqrt{x}}$

j $= 7(e^{4x} + 2)^6 \times 4e^{4x}$
$= 28e^{4x}(e^{4x} + 2)^6$

k $= -(5 + 4\sqrt{x})^{-2} \times 2x^{-\frac{1}{2}}$
$= \frac{-2}{\sqrt{x}(5 + 4\sqrt{x})^2}$

l $= 5(\frac{2}{x} - x)^4 \times (-2x^{-2} - 1)$
$= -5(\frac{2}{x^2} + 1)(\frac{2}{x} - x)^4$

7 **a** $\frac{dy}{dx} = 5(2x - 3)^4 \times 2$
SP: $10(2x - 3)^4 = 0$
$x = \frac{3}{2}$
$\therefore (\frac{3}{2}, 0)$

b $\frac{dy}{dx} = 3(x^2 - 4)^2 \times 2x$
SP: $6x(x^2 - 4)^2 = 0$
$x = 0$ or $x^2 = 4$
$x = 0, \pm 2$
$\therefore (-2, 0), (0, -64), (2, 0)$

c $\frac{dy}{dx} = 8 - 2e^{2x}$
SP: $8 - 2e^{2x} = 0$
$e^{2x} = 4$
$x = \frac{1}{2}\ln 4 = \ln 2$
$\therefore (\ln 2, 8\ln 2 - 4)$

d $\frac{dy}{dx} = \frac{1}{2}(1 + 2x^2)^{-\frac{1}{2}} \times 4x$
SP: $\frac{2x}{\sqrt{1 + 2x^2}} = 0$
$x = 0$
$\therefore (0, 1)$

e $\frac{dy}{dx} = \frac{2}{x - x^2} \times (1 - 2x)$
SP: $\frac{2(1 - 2x)}{x - x^2} = 0$
$x = \frac{1}{2}$
$\therefore (\frac{1}{2}, -4\ln 2)$

f $\frac{dy}{dx} = 4 - (x - 3)^{-2}$
SP: $4 - \frac{1}{(x-3)^2} = 0$
$(x - 3)^2 = \frac{1}{4},\ x - 3 = \pm\frac{1}{2}$
$x = \frac{5}{2}, \frac{7}{2}$
$\therefore (\frac{5}{2}, 8), (\frac{7}{2}, 16)$

8 **a** $x = 2 \ \therefore y = 1$
$\frac{dy}{dx} = 4(3x - 7)^3 \times 3 = 12(3x - 7)^3$
grad $= -12$
$\therefore y - 1 = -12(x - 2)$
$[\ y = 25 - 12x\ ]$

b $x = 0 \ \therefore y = 2$
$\frac{dy}{dx} = \frac{1}{1 + 4x} \times 4 = \frac{4}{1 + 4x}$
grad $= 4$
$\therefore y = 4x + 2$

c $x = 1 \ \therefore y = 3$
$\frac{dy}{dx} = -9(x^2 + 2)^{-2} \times 2x = -18x(x^2 + 2)^{-2}$
grad $= -2$
$\therefore y - 3 = -2(x - 1)$
$[\ y = 5 - 2x\ ]$

d $x = \frac{1}{4} \ \therefore y = \frac{1}{2}$
$\frac{dy}{dx} = \frac{1}{2}(5x - 1)^{-\frac{1}{2}} \times 5 = \frac{5}{2}(5x - 1)^{-\frac{1}{2}}$
grad $= 5$
$\therefore y - \frac{1}{2} = 5(x - \frac{1}{4})$
$[\ y = 5x - \frac{3}{4}\ ]$

9 **a** $x = -2 \ \therefore y = -9$
$\frac{dy}{dx} = e^{4-x^2} \times (-2x) = -2x\,e^{4-x^2}$
grad $= 4 \ \therefore$ grad of normal $= -\frac{1}{4}$
$\therefore y + 9 = -\frac{1}{4}(x + 2)$
$[\ y = -\frac{1}{4}x - \frac{19}{2}\ ]$

b $x = \frac{1}{2} \ \therefore y = \frac{1}{8}$
$\frac{dy}{dx} = 3(1 - 2x^2)^2 \times (-4x) = -12x(1 - 2x^2)^2$
grad $= -\frac{3}{2} \ \therefore$ grad of normal $= \frac{2}{3}$
$\therefore y - \frac{1}{8} = \frac{2}{3}(x - \frac{1}{2})$
$[\ 16x - 24y - 5 = 0\ ]$

c $x = 1 \ \therefore y = \frac{1}{2}$
$\frac{dy}{dx} = -(2 - \ln x)^{-2} \times (-\frac{1}{x}) = \frac{1}{x(2 - \ln x)^2}$
grad $= \frac{1}{4} \ \therefore$ grad of normal $= -4$
$\therefore y - \frac{1}{2} = -4(x - 1)$
$[\ y = \frac{9}{2} - 4x\ ]$

d $x = 3 \ \therefore y = 6e$
$\frac{dy}{dx} = 2e^{\frac{x}{3}}$
grad $= 2e \ \therefore$ grad of normal $= -\frac{1}{2e}$
$\therefore y - 6e = -\frac{1}{2e}(x - 3)$
$[\ x + 2ey - 12e^2 - 3 = 0\ ]$

1 $x = \frac{1}{2}$ $\therefore y = \frac{1}{4}$

$\frac{dy}{dx} = 2x + \frac{1}{4x-1} \times 4 = 2x + \frac{4}{4x-1}$

grad = 1 + 4 = 5

$\therefore y - \frac{1}{4} = 5(x - \frac{1}{2})$

$[\, y = 5x - \frac{9}{4} \,]$

2 **a** $\sqrt{8 - e^{2x}} = 2$

$8 - e^{2x} = 4$

$x = \frac{1}{2}\ln 4 = \ln 2$

b $\frac{dy}{dx} = \frac{1}{2}(8 - e^{2x})^{-\frac{1}{2}} \times (-2e^{2x})$

$= \frac{-e^{2x}}{\sqrt{8 - e^{2x}}}$

grad = –2

$\therefore y - 2 = -2(x - \ln 2)$

$2x + y = 2 + 2\ln 2$

$2x + y = 2 + \ln 2^2$

$2x + y = 2 + \ln 4$

3 **a** $\frac{dy}{dx} = 2 + \frac{1}{4-2x} \times (-2) = 2 - \frac{1}{2-x}$

$\frac{d^2y}{dx^2} = (2 - x)^{-2} \times (-1) = \frac{-1}{(2-x)^2}$

b SP: $2 - \frac{1}{2-x} = 0$

$2 - x = \frac{1}{2}$

$x = \frac{3}{2}$ $\therefore (\frac{3}{2}, 4)$

c $x = \frac{3}{2}$, $\frac{d^2y}{dx^2} = -4$ $\therefore$ maximum

4 **a** $\frac{dy}{dx} = -3(2x + 1)^{-2} \times 2 = \frac{-6}{(2x+1)^2}$

$x = 1$, grad $= -\frac{2}{3}$, $\therefore$ grad of normal $= \frac{3}{2}$

$\therefore y - 1 = \frac{3}{2}(x - 1)$

$[\, y = \frac{3}{2}x - \frac{1}{2} \,]$

b at Q $\frac{3x-1}{2} = \frac{3}{2x+1}$

$(3x - 1)(2x + 1) = 6$

$6x^2 + x - 7 = 0$

$(6x + 7)(x - 1) = 0$

$x = 1$ (at P) or $-\frac{7}{6}$

$\therefore Q\,(-\frac{7}{6}, -\frac{9}{4})$

5 **a** $t = 0, N = 20$ $\therefore a = 20$

$t = 8, N = 60$ $\therefore 60 = 20e^{8k}$

$k = \frac{1}{8}\ln 3 = 0.137$ (3sf)

b $N = 20e^{0.1373t}$

$t = 12, N = 104$ (3sf)

c $\frac{dN}{dt} = 20 \times 0.1373e^{0.1373t} = 2.747e^{0.1373t}$

$t = 12$, $\frac{dN}{dt} = 14.3$

$\therefore$ N increasing at 14.3 per second (3sf)

6 **a** $= 3(5 - 2x^2)^2 \times (-4x)$

$= -12x(5 - 2x^2)^2$

b SP: $-12x(5 - 2x^2)^2 = 0$

$x = 0$ or $x^2 = \frac{5}{2}$

$x = 0, \pm\frac{1}{2}\sqrt{10}$

$\therefore (-\frac{1}{2}\sqrt{10}, 0), (0, 125), (\frac{1}{2}\sqrt{10}, 0)$

c $x = \frac{3}{2}$, $y = \frac{1}{8}$

grad $= -18 \times \frac{1}{4} = -\frac{9}{2}$

$\therefore y - \frac{1}{8} = -\frac{9}{2}(x - \frac{3}{2})$

$8y - 1 = -36x + 54$

$36x + 8y - 55 = 0$

7 **a** $\frac{dy}{dx} = 4 - e^{2x}$

SP: $4 - e^{2x} = 0$

$x = \frac{1}{2}\ln 4 = \ln 2$

$\therefore$ $(\ln 2, 4\ln 2 - 2)$

b $\frac{d^2y}{dx^2} = -2e^{2x}$

$x = \ln 2$: $\frac{d^2y}{dx^2} = -8$ $\therefore$ maximum

8 **a** $f'(x) = \frac{3}{x} - 2$

b grad of curve = 4

$\therefore$ $\frac{3}{x} - 2 = 4$

$x = \frac{1}{2}$

c SP: $\frac{3}{x} - 2 = 0$

$x = \frac{3}{2}$ $\therefore$ $(\frac{3}{2}, 3\ln\frac{15}{2} - 3)$

d $x \geq \frac{3}{2}$

9 **a** $\frac{dy}{dx} = \frac{1}{2}(x^2+3)^{-\frac{1}{2}} \times 2x = \frac{x}{\sqrt{x^2+3}}$

at A, grad = $-\frac{1}{2}$

$\therefore$ $y - 2 = -\frac{1}{2}(x + 1)$

$[\, y = \frac{3}{2} - \frac{1}{2}x \,]$

b at B, grad = $\frac{1}{2}$

$\therefore$ grad of normal = -2

$\therefore$ $y - 2 = -2(x - 1)$

$[\, y = 4 - 2x \,]$

c $\frac{3}{2} - \frac{1}{2}x = 4 - 2x$

$x = \frac{5}{3}$

10 **a** 80°C

b 20°C, as $t \to \infty$, $T \to 20$

c $30 = 20 + 60e^{-25k}$

$e^{-25k} = \frac{30-20}{60} = \frac{1}{6}$

$k = \frac{-1}{25}\ln\frac{1}{6} = 0.0717$ (3sf)

d $T = 20 + 60e^{-0.07167t}$

$\frac{dT}{dt} = 60 \times (-0.07167)e^{-0.07167t}$

$= -4.300e^{-0.07167t}$

$t = 40$, $\frac{dT}{dt} = -0.245$

$\therefore$ temp. decreasing at 0.245°C min^{-1} (3sf)

11 **a** $f'(x) = 2x - 7 + \frac{4}{x} = 0$

$2x^2 - 7x + 4 = 0$

$x = \frac{7 \pm \sqrt{49-32}}{4} = \frac{7 \pm \sqrt{17}}{4}$

$x = 0.72, 2.78$

b $x = 2$ $\therefore$ $y = -10$, grad = -1

$\therefore$ $y + 10 = -(x - 2)$

$[\, y = -x - 8 \,]$

12 **a** $\frac{dy}{dx} = 2x + 8(x-1)^{-2}$

SP: $2x + \frac{8}{(x-1)^2} = 0$

$2x(x-1)^2 + 8 = 0$

$2x(x^2 - 2x + 1) + 8 = 0$

$2x^3 - 4x^2 + 2x + 8 = 0$

$x^3 - 2x^2 + x + 4 = 0$

b let $f(x) = x^3 - 2x^2 + x + 4$

$f(1) = 4$, $f(2) = 6$, $f(-1) = 0$

$\therefore$ $(x + 1)$ is a factor

$\therefore$ $(x + 1)(x^2 - 3x + 4) = 0$

$x = -1$ or $x^2 - 3x + 4 = 0$

$b^2 - 4ac = 9 - 16 = -7$

$b^2 - 4ac < 0$ $\therefore$ no real roots

$\therefore$ exactly one SP

$(-1, 5)$

c $\frac{d^2y}{dx^2} = 2 - 16(x-1)^{-3}$

when $x = -1$, $\frac{d^2y}{dx^2} = 4$

$\frac{d^2y}{dx^2} > 0$ $\therefore$ minimum