1 **a** $= [2x^2 - x]_1^3$

$= (18 - 3) - (2 - 1)$

$= 14$

b $= [x^3 + 2x]_0^1$

$= (1 + 2) - (0)$

$= 3$

c $= [\frac{1}{2}x^2 - \frac{1}{3}x^3]_0^3$

$= (\frac{9}{2} - 9) - (0)$

$= -\frac{9}{2}$

d $= \int_2^3 (9x^2 + 6x + 1) \; dx$

$= [3x^3 + 3x^2 + x]_2^3$

$= (81 + 27 + 3) - (24 + 12 + 2)$

$= 73$

e $= [\frac{1}{3}x^3 - 4x^2 - 3x]_1^2$

$= (\frac{8}{3} - 16 - 6) - (\frac{1}{3} - 4 - 3)$

$= -12\frac{2}{3}$

f $= [8x - 2x^2 + x^3]_{-2}^4$

$= (32 - 32 + 64) - (-16 - 8 - 8)$

$= 96$

g $= [\frac{1}{4}x^4 - x^2 - 7x]_1^4$

$= (64 - 16 - 28) - (\frac{1}{4} - 1 - 7)$

$= 27\frac{3}{4}$

h $= [5x + \frac{1}{3}x^3 - x^4]_{-2}^{-1}$

$= (-5 - \frac{1}{3} - 1) - (-10 - \frac{8}{3} - 16)$

$= 22\frac{1}{3}$

i $= [\frac{1}{5}x^5 + 2x^3 - \frac{1}{2}x^2]_{-1}^2$

$= (\frac{32}{5} + 16 - 2) - (-\frac{1}{5} - 2 - \frac{1}{2})$

$= 23\frac{1}{10}$

2 $\int_1^4 (3x^2 + ax - 5) \; dx = [x^3 + \frac{1}{2}ax^2 - 5x]_1^4$

$= (64 + 8a - 20) - (1 + \frac{1}{2}a - 5) = 48 + \frac{15}{2}a$

$\therefore \; 48 + \frac{15}{2}a = 18$

$a = -4$

3 $\int_{-1}^k (3x^2 - 12x + 9) \; dx = [x^3 - 6x^2 + 9x]_{-1}^k$

$= (k^3 - 6k^2 + 9k) - (-1 - 6 - 9) = k^3 - 6k^2 + 9k + 16$

$\therefore \; k^3 - 6k^2 + 9k + 16 = 16$

$k(k^2 - 6k + 9) = 0$

$k(k - 3)^2 = 0$

$k \neq 0 \; \therefore \; k = 3$

4 **a** $= \int_1^3 (2 - x^{-2}) \; dx$

$= [2x + x^{-1}]_1^3$

$= (6 + \frac{1}{3}) - (2 + 1)$

$= \frac{10}{3}$

b $= \int_{-2}^{-1} (6x + 4x^{-3}) \; dx$

$= [3x^2 - 2x^{-2}]_{-2}^{-1}$

$= (3 - 2) - (12 - \frac{1}{2})$

$= -10\frac{1}{2}$

c $= [2x^{\frac{3}{2}} - 4x]_1^4$

$= (16 - 16) - (2 - 4)$

$= 2$

d $= \int_{-1}^2 (2x^3 - \frac{1}{2}) \; dx$

$= [\frac{1}{2}x^4 - \frac{1}{2}x]_{-1}^2$

$= (8 - 1) - (\frac{1}{2} + \frac{1}{2})$

$= 6$

e $= [\frac{1}{2}x^2 - \frac{3}{2}x^{\frac{2}{3}}]_1^8$

$= (32 - 6) - (\frac{1}{2} - \frac{3}{2})$

$= 27$

f $= \int_2^3 (\frac{1}{3}x^{-2} - 2x) \; dx$

$= [-\frac{1}{3}x^{-1} - x^2]_2^3$

$= (-\frac{1}{9} - 9) - (-\frac{1}{6} - 4)$

$= -4\frac{17}{18}$

5 $= \int_1^3 (3x^2 - 6x + 7) \; dx$

$= [x^3 - 3x^2 + 7x]_1^3$

$= (27 - 27 + 21) - (1 - 3 + 7) = 16$

6 **a** $= \int_0^2 (x^2 + 2) \ dx$

$= [\frac{1}{3}x^3 + 2x]_0^2$

$= (\frac{8}{3} + 4) - 0 = 6\frac{2}{3}$

b $= \int_{-2}^1 (3x^2 + 8x + 6) \ dx$

$= [x^3 + 4x^2 + 6x]_{-2}^1$

$= (1 + 4 + 6) - (-8 + 16 - 12) = 15$

c $= \int_2^4 (9 + 2x - x^2) \ dx$

$= [9x + x^2 - \frac{1}{3}x^3]_2^4$

$= (36 + 16 - \frac{64}{3}) - (18 + 4 - \frac{8}{3}) = 11\frac{1}{3}$

d $= \int_{-1}^0 (x^3 - 4x + 1) \ dx$

$= [\frac{1}{4}x^4 - 2x^2 + x]_{-1}^0$

$= 0 - (\frac{1}{4} - 2 - 1) = \frac{11}{4}$

e $= \int_1^4 (2x + 3x^{\frac{1}{2}}) \ dx$

$= [x^2 + 2x^{\frac{3}{2}}]_1^4$

$= (16 + 16) - (1 + 2) = 29$

f $= \int_{-5}^{-1} (3 + 5x^{-2}) \ dx$

$= [3x - 5x^{-1}]_{-5}^{-1}$

$= (-3 + 5) - (-15 + 1) = 16$

7 **a** $y = 0 \implies 4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$

$\therefore (-2, 0)$ and $(2, 0)$

b $= \int_{-2}^2 (4 - x^2) \ dx$

$= [4x - \frac{1}{3}x^3]_{-2}^2$

$= (8 - \frac{8}{3}) - (-8 + \frac{8}{3})$

$= 10\frac{2}{3}$

8 **a** $6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0, 2$
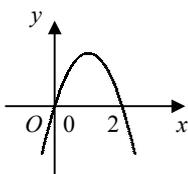
area =

$\int_0^2 (6x - 3x^2) \ dx$

$= [3x^2 - x^3]_0^2$

$= (12 - 8) - 0$

$= 4$

b $-x^2 + 4x - 3 = 0$

$-(x - 1)(x - 3) = 0$

$x = 1, 3$
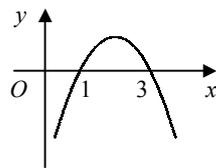
area =

$\int_1^3 (-x^2 + 4x - 3) \ dx$

$= [-\frac{1}{3}x^3 + 2x^2 - 3x]_1^3$

$= (-9 + 18 - 9)$

$\quad - (-\frac{1}{3} + 2 - 3)$

$= \frac{4}{3}$

c $4 - 3x - x^2 = 0$

$-(x + 4)(x - 1) = 0$

$x = -4, 1$
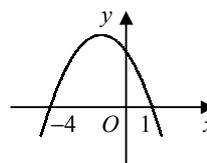

area =

$\int_{-4}^1 (4 - 3x - x^2) \ dx$

$= [4x - \frac{3}{2}x^2 - \frac{1}{3}x^3]_{-4}^1$

$= (4 - \frac{3}{2} - \frac{1}{3})$

$\quad - (-16 - 24 + \frac{64}{3})$

$= 20\frac{5}{6}$

d $2x^{\frac{1}{2}} - x = 0$

$x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0$

$x^{\frac{1}{2}} = 0, 2$

$x = 0, 4$
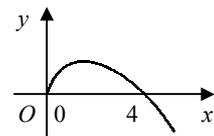
area =

$\int_0^4 (2x^{\frac{1}{2}} - x) \ dx$

$= [\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2]_0^4$

$= (\frac{32}{3} - 8) - 0$

$= \frac{8}{3}$

9   **a** $x^2 + 4x = 0$
         $x(x + 4) = 0$
         $x = -4, 0$
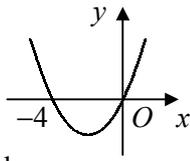
   **b** $= \int_0^2 (x^2 + 4x)\ dx$

         $= [\frac{1}{3}x^3 + 2x^2]_0^2$

         $= (\frac{8}{3} + 8) - 0 = 10\frac{2}{3}$

10   **a** $x^2 + 2x - 15 = 0$
         $(x + 5)(x - 3) = 0$
         $x = -5, 3$
         $\therefore\ (-5, 0)\ \text{and}\ (3, 0)$

     b $= [\frac{1}{3}x^3 + x^2 - 15x]_0^3$

         $= (9 + 9 - 45) - 0 = -27$

     c   27

11   **a** $x^3 - 5x^2 + 6x = 0$
         $x(x - 2)(x - 3) = 0$
         $x = 0, 2, 3$
         $\therefore\ (0, 0), (2, 0)\ \text{and}\ (3, 0)$

     b $\int_0^2 (x^3 - 5x^2 + 6x)\ dx$

         $= [\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2]_0^2$

         $= (4 - \frac{40}{3} + 12) - 0 = \frac{8}{3}$

         $\int_2^3 (x^3 - 5x^2 + 6x)\ dx$

         $= [\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2]_2^3$

         $= (\frac{81}{4} - 45 + 27) - \frac{8}{3} = -\frac{5}{12}$

         total area $= \frac{8}{3} + \frac{5}{12} = 3\frac{1}{12}$

12   **a** $x^2 - 3x + 4 = x + 1$
         $x^2 - 4x + 3 = 0$
         $(x - 1)(x - 3) = 0$
         $x = 1, 3$

         $\therefore\ (1, 2)\ \text{and}\ (3, 4)$

     b   area below curve

         $= \int_1^3 (x^2 - 3x + 4)\ dx$

         $= [\frac{1}{3}x^3 - \frac{3}{2}x^2 + 4x]_1^3$

         $= (9 - \frac{27}{2} + 12) - (\frac{1}{3} - \frac{3}{2} + 4) = \frac{14}{3}$

         area below line $= \frac{1}{2} \times 2 \times (2 + 4) = 6$

         shaded area $= 6 - \frac{14}{3} = \frac{4}{3}$

13   **a**
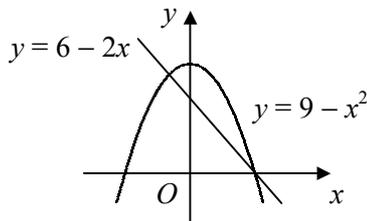
     $9 - x^2 = 6 - 2x$
     $x^2 - 2x - 3 = 0$
     $(x + 1)(x - 3) = 0$
     $x = -1, 3$
     $\therefore$ intersect at $(-1, 8)$ and $(3, 0)$
     area below curve

         $= \int_{-1}^3 (9 - x^2)\ dx$

         $= [9x - \frac{1}{3}x^3]_{-1}^3$

         $= (27 - 9) - (-9 + \frac{1}{3})$

         $= 26\frac{2}{3}$

     area below line

         $= \frac{1}{2} \times 4 \times 8 = 16$

     area between line and curve

         $= 26\frac{2}{3} - 16 = 10\frac{2}{3}$

     b
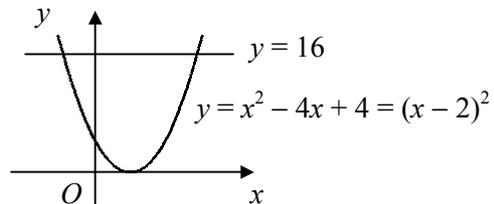
     $x^2 - 4x + 4 = 16$
     $x^2 - 4x - 12 = 0$
     $(x + 2)(x - 6) = 0$
     $x = -2, 6$
     $\therefore$ intersect at $(-2, 16)$ and $(6, 16)$
     area below curve

         $= \int_{-2}^6 (x^2 - 4x + 4)\ dx$

         $= [\frac{1}{3}x^3 - 2x^2 + 4x]_{-2}^6$

         $= (72 - 72 + 24) - (-\frac{8}{3} - 8 - 8)$

         $= 42\frac{2}{3}$

     area below line

         $= 8 \times 16 = 128$

     area between line and curve

         $= 128 - 42\frac{2}{3} = 85\frac{1}{3}$

c $y = x^2 - 5x - 6 \Rightarrow y = (x + 1)(x - 6)$
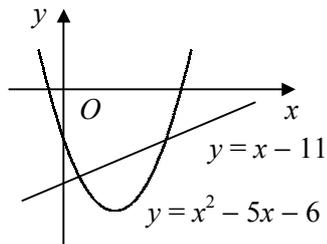
$x^2 - 5x - 6 = x - 11$

$x^2 - 6x + 5 = 0$

$(x - 1)(x - 5) = 0$
$x = 1, 5$
∴ intersect at $(1, -10)$ and $(5, -6)$
area above curve

$\quad = -\int_{1}^{5} (x^2 - 5x - 6) \ \mathrm{d}x$

$\quad = -[\frac{1}{3}x^3 - \frac{5}{2}x^2 - 6x]_{1}^{5}$

$\quad = -[(\frac{125}{3} - \frac{125}{2} - 30) - (\frac{1}{3} - \frac{5}{2} - 6)]$

$\quad = 42\frac{2}{3}$

area above line

$\quad = \frac{1}{2} \times 4 \times (10 + 6) = 32$

area between line and curve

$\quad = 42\frac{2}{3} - 32 = 10\frac{2}{3}$

d $x - 2y = 0 \Rightarrow y = \frac{1}{2}x$
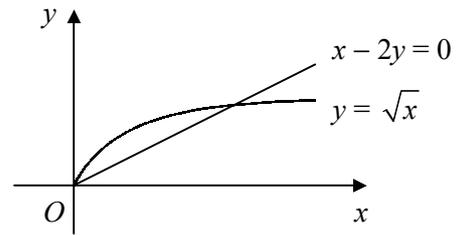
$x^{\frac{1}{2}} = \frac{1}{2}x$

$\frac{1}{2}x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0$

$x^{\frac{1}{2}} = 0, 2$
$x = 0, 4$
∴ intersect at $(0, 0)$ and $(4, 2)$
area below curve

$\quad = \int_{0}^{4} x^{\frac{1}{2}} \ \mathrm{d}x$

$\quad = [\frac{2}{3}x^{\frac{3}{2}}]_{0}^{4}$

$\quad = \frac{16}{3} - 0 = \frac{16}{3}$

area below line

$\quad = \frac{1}{2} \times 4 \times 2 = 4$

area between line and curve

$\quad = \frac{16}{3} - 4 = \frac{4}{3}$