1 **a** $= \log_{10} a + \log_{10} b$

b $= \log_{10} a + \log_{10} b^7$
$= \log_{10} a + 7\log_{10} b$

c $= \log_{10} a^3 - \log_{10} b$
$= 3\log_{10} a - \log_{10} b$

d $= \log_{10} a + \log_{10} b^{\frac{1}{2}}$
$= \log_{10} a + \frac{1}{2}\log_{10} b$

e $= 2\log_{10} ab$
$= 2\log_{10} a + 2\log_{10} b$

f $= -\log_{10} ab$
$= -\log_{10} a - \log_{10} b$

g $= \log_{10} a^{\frac{3}{2}} + \log_{10} b^{\frac{5}{2}}$
$= \frac{3}{2}\log_{10} a + \frac{5}{2}\log_{10} b$

h $= 3(\log_{10} a^2 - \log_{10} b^{\frac{1}{3}})$
$= 6\log_{10} a - \log_{10} b$

2 **a** $= \log_q 8^2$
$= 2y$

b $= \log_q 8^{\frac{1}{3}}$
$= \frac{1}{3}y$

c $= \log_q 16 - \log_q q$
$= \log_q 8^{\frac{4}{3}} - 1$
$= \frac{4}{3}y - 1$

d $= \log_q 4 + \log_q q^3$
$= \log_q 8^{\frac{2}{3}} + 3$
$= \frac{2}{3}y + 3$

3 **a** $= \lg(2 \times 3^2)$
$= \lg 2 + 2\lg 3$
$= a + 2b$

b $= \lg(2^5 \times 3)$
$= 5\lg 2 + \lg 3$
$= 5a + b$

c $= \lg 9 - \lg 16$
$= \lg 3^2 - \lg 2^4$
$= 2\lg 3 - 4\lg 2$
$= 2b - 4a$

d $= \lg(2 \times 3) - \lg 2^3$
$= \lg 2 + \lg 3 - 3\lg 2$
$= \lg 3 - 2\lg 2$
$= b - 2a$

e $= \frac{1}{2}\lg 6$
$= \frac{1}{2}(\lg 2 + \lg 3)$
$= \frac{1}{2}(a + b)$

f $= \frac{3}{2}\lg 2^4 + \frac{1}{2}\lg 3^4$
$= 6\lg 2 + 2\lg 3$
$= 6a + 2b$

g $= 4\lg 3 - 3(\lg 2 + \lg 3)$
$= \lg 3 - 3\lg 2$
$= b - 3a$

h $= \lg(6\times10) + \lg(2\times10) - 2$
$= \lg 6 + 1 + \lg 2 + 1 - 2$
$= \lg 2 + \lg 3 + \lg 2$
$= 2a + b$

4 **a** $= \log_5 10 - \log_5 2$
$= \log_5 5$
$= 1$

b $= \log_{12} 16 + \log_{12} 9$
$= \log_{12} 144$
$= 2$

c $= \log_4 8$
$= \log_4 4^{\frac{3}{2}}$
$= \frac{3}{2}$

d $= \dfrac{\log_7 3^4}{\log_7 3}$
$= \dfrac{4\log_7 3}{\log_7 3}$
$= 4$

e $= \log_{27} \dfrac{12^3}{72^2}$
$= \log_{27} \dfrac{12\times12\times12}{6\times12\times6\times12}$
$= \log_{27} \frac{1}{3} = -\frac{1}{3}$

f $= \dfrac{\log_{11} 5^2}{-\log_{11} 5}$
$= \dfrac{2\log_{11} 5}{-\log_{11} 5}$
$= -2$

5 **a** $x = 3^{1.8}$
$x = 7.22$

b $x = 5^{-0.3}$
$x = 0.617$

c $x - 3 = 8^{2.1}$
$x = 3 + 8^{2.1}$
$x = 81.8$

d $\frac{1}{2}x + 1 = 4^{3.2}$
$x = 2(4^{3.2} - 1)$
$x = 167$

e $\log_2 3y = 5.3$
$3y = 2^{5.3}$
$y = \frac{1}{3} \times 2^{5.3}$
$y = 13.1$

f $\log_6(1 - 5t) = -0.6$
$1 - 5t = 6^{-0.6}$
$t = \frac{1}{5}(1 - 6^{-0.6})$
$t = 0.132$

6 **a** $= \log_2 x^5$

b $= \log_2(x^2 + 4x)$

c $= \log_2 x^2 + \log_2 x$
$= \log_2 x^3$

d $= \log_2(x - 2)^3 - \log_2 x^4$
$= \log_2 \dfrac{(x-2)^3}{x^4}$

e $= \log_2 \dfrac{x^2 - 1}{x + 1}$
$= \log_2 \dfrac{(x+1)(x-1)}{x+1}$
$= \log_2(x - 1)$

f $= \log_2 x - 2\log_2 x + \frac{2}{3}\log_2 x$
$= -\frac{1}{3}\log_2 x$
$= \log_2 x^{-\frac{1}{3}}$

7 **a** $\log_3 5x = \log_3 (2x + 3)$

$5x = 2x + 3$
$x = 1$

b $\log_9 10x = \frac{3}{2}$

$10x = 9^{\frac{3}{2}} = 27$
$x = 2.7$

c $\log_4 \frac{x}{x-1} = \log_4 3 + \log_4 2 = \log_4 6$

$\frac{x}{x-1} = 6$
$x = 6x - 6$
$x = \frac{6}{5}$

d $\log_5 \frac{5x}{x+2} = \log_5 \frac{x+6}{x}$

$\frac{5x}{x+2} = \frac{x+6}{x}$
$5x^2 = (x + 2)(x + 6) = x^2 + 8x + 12$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1, 3$
$\log_5 x$ not real for $x = -1$ $\therefore$ $x = 3$

e $\log_6 x^2 = \log_6 5(2x - 5)$

$x^2 = 5(2x - 5)$
$x^2 - 10x + 25 = 0$
$(x - 5)^2 = 0$
$x = 5$

f $\log_7 4x - \log_7 \frac{1}{x-6} = 1$

$\log_7 4x(x - 6) = 1$
$4x(x - 6) = 7$
$4x^2 - 24x - 7 = 0$
$x = \frac{24 \pm \sqrt{576+112}}{8} = 3 \pm \frac{1}{2}\sqrt{43}$
$\log_7 4x$ not real for $x = 3 - \frac{1}{2}\sqrt{43}$
$\therefore$ $x = 3 + \frac{1}{2}\sqrt{43}$ [= 6.28 (3sf)]

8 **a** $\log_x y = 2 \Rightarrow y = x^2$

sub. $x^3 = 27$
$x = 3$
$\therefore$ $x = 3, y = 9$

b $\log_5 x - 2\log_5 y = \log_5 2 \Rightarrow \frac{x}{y^2} = 2$
$\Rightarrow x = 2y^2$

sub. $3y^2 = 12$
$y^2 = 4$
for real $\log_5 y$, $y > 0$ $\therefore$ $y = 2$
$\therefore$ $x = 8, y = 2$

c $\log_y 32 = -\frac{5}{2} \Rightarrow y^{-\frac{5}{2}} = 32$
$\Rightarrow y = 32^{-\frac{2}{5}} = \frac{1}{4}$

sub. $\log_2 x = 3 - 2\log_2 \frac{1}{4}$
$\log_2 x = 3 - (-4) = 7$
$x = 2^7 = 128$
$\therefore$ $x = 128, y = \frac{1}{4}$

d $\log_y x = \frac{3}{2} \Rightarrow y^{\frac{3}{2}} = x$
$\Rightarrow y^{\frac{1}{2}} = x^{\frac{1}{3}}$

sub. $4x^{\frac{1}{3}} = 20$
$x^{\frac{1}{3}} = 5$
$x = 5^3 = 125$
$\therefore$ $x = 125, y = 25$

e $\log_a x + \log_a 3 = \frac{1}{2}\log_a y \Rightarrow 3x = y^{\frac{1}{2}}$
$\Rightarrow y = 9x^2$

sub. $3x + 9x^2 = 20$
$9x^2 + 3x - 20 = 0$
$(3x + 5)(3x - 4) = 0$
for real $\log_a x$, $x > 0$ $\therefore$ $x = \frac{4}{3}$
$\therefore$ $x = \frac{4}{3}, y = 16$

f $\log_{10} y + 2\log_{10} x = 3 \Rightarrow x^2 y = 10^3$
$\log_2 y - \log_2 x = 3 \Rightarrow \frac{y}{x} = 2^3$
$\Rightarrow y = 8x$

sub. $8x^3 = 1000$
$x^3 = 125$
$x = 5$
$\therefore$ $x = 5, y = 40$